

Recommended
auction model for the
award of 700, 1400
and 2100 MHz
spectrum

Prepared for the Dutch
Ministry of Economic Affairs

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Executive Summary

We have been asked by the Dutch Ministry of Economic Affairs (the Ministry) to recommend an auction model and propose detailed rules for the upcoming award of spectrum in the 700, 1400 and 2100 MHz bands (the Award), where the following spectrum will be offered:

- 2x30 MHz of FDD spectrum in the 700 MHz band;
- 40 MHz of SDL spectrum in the 1400 MHz band; and
- 2x60 MHz of FDD spectrum in the 2100 MHz band.

The objectives set for the auction design are:

- ensuring an efficient assignment of spectrum;
- generating a realistic revenue for the government;
- simplicity;
- transparency; and
- freedom of choice (meaning that all bidders should be able to express their demand in their bids on a level playing field).

The Ministry expects participation from at least the three MNOs (following the merger of T-Mobile and Tele 2). Some other parties expressed interest in acquiring some spectrum, especially in the 2100 MHz band (possible for regional or offshore use).

Following the recommendations of the Dutch competition Authority ACM, the Ministry has decided that following spectrum caps shall apply to each individual operator (considering existing spectrum holdings):

- 40% on all mobile spectrum; and
- 40% on all mobile spectrum below 1 GHz, rounded up to the nearest multiple of 10 MHz (2x5 MHz).

Given differences in the spectrum currently held by the three MNOs, the maximum amount of spectrum that each MNO can acquire in the auction under these caps varies.

We understand that there are no known systematic value differences across the blocks within each band. Therefore, we assume that the spectrum will be offered initially as frequency-generic lots within categories for each band, with the assignment of specific frequencies determined in a follow-up stage. We also assume that frequency-generic lots will have a small size, of 2x5 MHz for FDD spectrum and 5 MHz for SDL spectrum. We note that whilst a larger lots size for 700 MHz was also considered by Aetha with a view to mitigating aggregation risks, this would severely limit the range of possible assignments and the scope for competition for spectrum in that band, and would impose tighter constraints than those that

arise from the spectrum caps on some operators. Given that stakeholders have stated that value synergies within and across bands are limited, and aggregation risks are not material, and given that there is evidence from other awards that some MNOs have been willing to acquire single 2x5 MHz lots, we consider that the smaller size of 2x5 MHz is preferable on the grounds of the flexibility to support a wider range of outcomes in line with the spectrum caps and the greater scope for competition it provides. This both supports efficiency and increases freedom of choice.

In order to assess alternative auction formats for this award, we set out a number of requirements that need to be met to achieve objectives set out by the Ministry for the award, including a number of different aspects that the format should address in order to promote an efficient assignment.

For this award, based on the views put forward by stakeholders, we consider that aggregation risks are not of any substantial concern, and that therefore the main risks that the auction format should address to mitigate the risk of inefficiencies are as follows:

- substitution risks (where a bidder might end up winning some lots that are not its preferred lots given the final auction prices, and which may arise due to uncertainty about competitors' demand and/or switching impediments);
- distorted bids due to strategic complexity (where bidders may be unable to directly reflect their demand in their bids and may need to consider their expectations about the outcome when determining their bids);
- distorted bids due to strategic bidding (where bidders could gain from reflecting their true demand through bids, through bid strategies aimed at keeping prices low or inflicting greater costs on competitors); and
- bid errors arising from mechanical complexity (if the auction rules are complex and bidders fail to correctly anticipate the consequences of their actions, or simply make mistakes when preparing their bids).

Given these requirements, we consider that:

- an open, multi-round process is better suited to meeting the Ministry's objectives than a sealed-bid process, as this will mitigate uncertainties faced by bidders and reduce strategic complexity; and
- a non-combinatorial format seems better suited for this award, as the additional mechanical complexity of combinatorial auction models does not appear to be justified given that aggregation risks are not considered to be a material issue for this award.

Given this, we shortlist the SMRA-based formats (which include the SMRA, SMRA with augmented switching, SMRA clock hybrid, Clock-Plus) as the best candidate formats for this award. Being pay-as-bid formats, all of them are simple and

provide bidders with maximum certainty, but also create incentives for bidders strategically to reduce demand in order to benefit from lower prices. However, we consider that the potential risk of strategic demand reduction can be addressed without having to forego the benefits of simple pay-as-bid formats by avoiding excessively low reserve prices. Setting reserve prices that are closer to the prices that could be expected in a competitive auction will curtail the potential benefits from reducing demand too early and thus curb the incentives for strategic demand reduction.

For the avoidance of doubt, we note that even though the level of reserve prices can affect the extent to which various objectives (specifically achieving a realistic revenue) will be met, our recommendation for an SMRA-based format does not depend on reserve prices.

Of these various formats, we recommend the SMRA clock hybrid, on the basis that:

- it is more efficient when there are many identical lots, where the standard SMRA or the SMRA with Augmented Switching could take an unreasonably long time to resolve; and
- it is more transparent and is both mechanically and strategically simpler than the Clock-Plus – the use of standing high bids contributes to the predictability of the format and provides a reliable framework for bidders to adjust their bids in response to the evolution of prices and demand.

Only if – contrary to the views expressed by stakeholders to date – aggregation risks were material would we recommend using a combinatorial auction format. In this case, we would suggest using a CMRA format, as this would effectively mitigate aggregation risks without creating a risk of price asymmetries arising from the use of a second price rule when bidders have different limits with respect to the maximum amount of spectrum for which they can bid.

This report also provides specific auction rules for the recommended auction format.

1 Introduction

Scope of our advice

We have been asked by the Dutch Ministry of Economic Affairs (the Ministry) to recommend an auction model and propose detailed rules for the upcoming award of spectrum in the 700, 1400 and 2100 MHz bands (the Award).

The available spectrum comprises:

- 2x30 MHz of FDD spectrum in the 700 MHz band;
- 40 MHz of SDL spectrum in the 1400 MHz band; and
- 2x60 MHz of FDD spectrum in the 2100 MHz band.¹

The recommendation and the specific rules:

- are based on the auction objectives expressed by the Ministry in our initial instructions and further clarified in a note from the Ministry of 20 June 2019 and the competition safeguards adopted by the Ministry following the proposals from the Dutch competition authority ACM; and
- also consider the likely level and structure of demand for the spectrum and underlying technical considerations, in relation to potential valuation synergies within as well as across bands.

In preparing our recommendation, we have drawn on:

- materials previously prepared or collected in relation to the Award (the report on potential linkages between the three bands prepared by Aetha² and the initial recommendations on an auction model put forward by CREED³);
- the views expressed by industry participants in their responses to various informal consultations, including an auction workshop held by the Ministry in March 2017⁴ and the National Frequency Policy Forum in 2018, where our initial views on suitable auction models were presented⁵; and
- the “Addendum to assignment on advice for Multi-band auction (700, 1400, 2100 MHz)” from the Ministry, dated 20

¹ We understand that the duplex gap in the 700 MHz band and the extension bands of the 1400 MHz band as well as the TDD spectrum in the 2100 MHz band are not to be included. We also understand that there is no intention at present to include spectrum in the 3500 MHz band.

² Aetha, Research into linkages between the 700MHz, 1452-1492MHz and 2100MHz bands; Final report for Ministry of Economic Affairs, 7 October 2016

³ CREED, Rapport Veilingontwerp 700/1400/2100 MHz Vergunningen, January 2018

⁴ Veilingseminar 14 maart 2017 bij het ministerie van Economische Zaken

⁵ Voorbereiding multibandveiling 700, 1400 en 2100 MHz vergunningen (<https://www.rijksoverheid.nl/onderwerpen/telecommunicatie/documenten/publicaties/2018/10/09/voorbereiding-multibandveiling-700-1400-en-2100-mhz-vergunningen>)

June 2019, which clarifies how the Dutch Government assesses the change in market conditions after the merger of T-Mobile and Tele2 and asks for specific attention to be given to the risks that to the objective of realizing a realistic revenues that have been created or amplified by the merger leaving three equally well placed entities to compete for an amount of spectrum that lends itself to being shared equally amongst three parties.

Structure of this report

The remainder of this report is structured as follows:

- In the next section, we summarise our understanding of the context of the Award, looking at the auction objectives, the proposed competition safeguards and the likely structure of valuations given a packaging of the available spectrum into individual lots that best promotes the underlying objectives, and set out the key assumptions we have distilled from this.
- Section 3 discusses the requirements for a suitable auction format that follow from the underlying objectives and develops a recommendation for a design considering a range of candidate auction models.
- Section 4 then sets out detailed aspects of the auction rules such as the activity rules and the appropriate information policy, as well as the process for the assignment of specific frequencies.

A set of draft auction rules is provided in an Annex.

2 Background and key assumptions

Auction objectives The Ministry primarily wants the auction to **produce an efficient allocation of the available spectrum**. This means that spectrum should end up with those users who can create the greatest value in effectively competitive downstream markets. Achieving this objective requires an auction in which all potential auction participants can express their requirements in competition with each other and have a realistic chance of winning. We understand this to imply that the design should not favour specific bidders or outcomes but should give all bidders the opportunity to bid for spectrum on a level playing field (which is explicitly stated in a further objective set out by the Ministry). Subject to promoting an efficient allocation of spectrum, the auction should also **generate a realistic revenue** for the government.

As we will discuss in more detail below, the overarching efficiency objective has a number of implications for the appropriate auction design, such as discouraging strategic bidding (which may prevent an efficient allocation) or protecting bidders from aggregation risks where there are material synergies in the underlying valuations.

Within these overarching objectives of achieving an efficient allocation, the Ministry has set **additional goals for the auction design**, namely that the it should:

- **be simple**, in terms of both auction rules and mechanics and bid strategy and decisions;
- **be transparent**; and
- provide **freedom of choice**, meaning that all bidders should be able to express their demand for spectrum on a level-playing field.

As noted above, we consider that the last of these goals is closely linked to the primary objective of achieving an efficient outcome, as ensuring that bidders can express their demand through the bids they place in competition with each other is essential to ensure the efficient allocation of scarce frequency resources.

Participation After the completion of the merger of T-Mobile and Tele 2, the Ministry expects **at least three participants** in the auction, namely:

- KPN;
- VodafoneZiggo; and
- the merged T-Mobile/Tele2.

*Measures to
safeguard
competition*

Some other parties expressed interest in acquiring spectrum, especially in the 2100 MHz band⁶ (possibly for use on a regional basis or offshore), and there may be additional interest from further parties.

Following ACM's recommendations, the Ministry has decided to apply the following caps of relevance for this award:

- a cap of 40% on all spectrum designated for mobile communications, which currently⁷ includes the 700, 800, 900, 1400, 1800, 2100, and 2600 MHz bands – the 'overall cap'; and
- a cap of 40% on all spectrum below 1 GHz designated for mobile communications, i.e. the 700, 800 and 900 MHz bands, rounded up to the nearest multiple of 10 MHz (2x5 MHz) – the 'sub-1 GHz cap'.

Given that the spectrum holdings of the three MNOs in the bands, excluding frequencies that are currently assigned but will be included in the award, are asymmetric (as shown in Table 1), this implies that different bidders will face different constraints on the amount of bandwidth that they can acquire in the auction.⁸

⁶ Interest from such parties in these specific bands was largely driven by expectations that FDD spectrum in the 700 MHz band would be relatively expensive (Aetha, *op. cit.*).

⁷ In the future spectrum designated for mobile communications will include spectrum in the 3.5 GHz band (when that band becomes available) and potentially any other spectrum below 6 GHz licensed on a national basis.

⁸ As part of implementing ACM's advice the Ministry has to decide on the possible exclusion of some existing spectrum holdings in the 2600 MHz band when calculating the total amount of spectrum available and the individual holdings. This could result in a small change of the total amount of spectrum available and the spectrum holdings of some operators, which in turn could lead to a minor change in the amount of bandwidth (5 MHz) that some of the MNOs can acquire in the auction. Such a small change would not have any impact on our recommendations.

Table 1: Spectrum holdings of the three MNOs (MHz) and implied constraints

Band	T-Mobile/ Tele2	VodafoneZiggo	KPN	TOTAL
800 MHz	20	20	20	60
900 MHz	30	20	20	70
1800 MHz	60	40	40	140
2600 MHz FDD	50	60	20	130
2600 MHz TDD	25		25	50
Total below 1GHz	50	40	40	130
Total	185	140	125	450

The addition of the 700 MHz band implies that the total amount of spectrum below 1 GHz increases to 190 MHz. The total amount of spectrum including the three bands available for the award will be $450 + 60 + 40 + 120 = 670$ MHz.

Under these caps, no operator will be allowed to hold more than 80 MHz (2x40 MHz of paired spectrum) below 1 GHz, and no more than 268 MHz (which in this case effectively means 265 MHz) of mobile spectrum overall.

Table 2 shows the implied limits that apply to existing operators bidding in the auction.

Table 2: Implied limits on the amount of spectrum that MNOs can acquire in the auction

Band	T-Mobile/ Tele2	VodafoneZiggo	KPN
700 MHz	30 MHz (3 blocks)	40 MHz (4 blocks)	40 MHz (4 blocks)
Total	80 MHz	125 MHz	140 MHz

Spectrum packaging and the structure of valuations

We understand that spectrum blocks within each band are of very similar value so that they can be offered initially as frequency-generic blocks with a subsequent assignment of specific frequencies to winners of bandwidth. This approach, which is well tested and has been used in several spectrum auctions, including previous awards in the Netherlands, guarantees that winners will receive contiguous assignments. It greatly simplifies the process for bidders who can focus on the amount of spectrum they wish to acquire without needing to be concerned about the risk of fragmented assignments while still

being able to express their preferences over specific frequency ranges within a band.

Reflecting the overarching efficiency objective and given that fragmentation risks are removed when offering lots as frequency-generic blocks, the available frequencies will be offered in small blocks. This allows operators to assemble their preferred spectrum portfolios in response to relative prices that reflect competing demands for different bands. The minimum size of individual blocks is determined by technical requirements and we assume that, in line with standard international practice, the spectrum available will be packaged into blocks of 2x5 MHz for FDD spectrum and 5 MHz for SDL spectrum.

This approach to spectrum packaging has been used in previous auctions in the Netherlands and broadly matches Aetha's recommendations. However, Aetha has also considered offering the 700 MHz band as blocks of 2x10 MHz lots, based on the presumption that offering smaller blocks could lead to aggregation risks.⁹ Aetha's argues that offering larger blocks might be a way of reducing the complexity of the auction design, based on the assumption that "*a minimum assignment of 2x10 MHz is required to make efficient use of the spectrum*" and the design therefore would need to make sure that "*no organisation that wins spectrum in this band is left with less than a minimum of 2x10 MHz*".¹⁰ Specifically, Aetha suggests that the spectrum in this band could be "*pre-packaged into three lots of 2x10 MHz, with each organisation only able to acquire a maximum of one of these lots, seeing it is highly likely that this will be the eventual outcome of the auction in any case.*"¹¹

We note that under the spectrum caps, two bidders would be able to bid for more than one 2x10 MHz block, whilst T-Mobile/Tele2, so that Aetha's proposal of capping each bidder to one block would be substantially more restrictive than the caps adopted by the Ministry. Though it would be possible to use the larger block size without necessarily pre-determining the outcome as suggested by Aetha, this would still mean that T-Mobile/Tele2 effectively faces a tighter limit than implied by the sub-1 GHz cap (as it could only bid for 20 rather than 30 MHz they it is able to acquire under the cap). Moreover, with these larger blocks, any asymmetric result would imply that one

⁹ The risk for operators with increasing marginal valuations of spectrum (i.e. for whom the value of spectrum portfolios grows disproportionately with size) is that, if they win less spectrum than they bid for, they might end up overpaying for the amount of spectrum they win. This risk can be mitigated through auction design, e.g. through using package bidding in general or through using some form of guarantee that bidders will only win a minimum endowment when the risks are particularly relevant with respect to obtaining a minimum bandwidth required for a viable business case.

¹⁰ Aetha, *op cit*, p 15.

¹¹ *Ibid.*

of the bidders will not obtain any 700 MHz spectrum, which may be inefficient.

Even though operators may *prefer* to acquire a minimum of 2x10 MHz rather than just 2x5 MHz of spectrum in the 700 MHz band, it is not clear that 2x10 MHz is the *minimum usable bandwidth* in this band. The outcome of the recent Swiss auction, in which Sunrise acquired a single 2x5 MHz block of 700 MHz spectrum, or the Danish auction, in which TT network ended up buying a single block suggests that 2x5 MHz can be usefully deployed and can be attractive for bidders.

More generally, we understand that stakeholders have indicated that valuation synergies are limited and that aggregation risks are therefore not material, both within bands and across the different bands available.¹²

Given this, we consider that valuation synergies are limited, and aggregation risks are manageable by bidders without the need to impose measures that would eliminate any outcome in which a winner fails to obtain a certain minimum spectrum endowment.

¹² Even though complementarities within a given band arise naturally because of spectral efficiency gains from deploying wider carriers with spread spectrum technologies, the practical significance of such complementarities appears to be limited. Given this, we conclude that any residual aggregation risks within bands should be manageable. In this regard it is instructive to note that Aetha considers that in the 2100 MHz band “*in the event that this requirement for a minimum acquisition of 2x10MHz resulted in a disproportionate amount of complexity in the auction, it could be withdrawn as the valuations of spectrum by bidders will probably lead to a minimum block size of 2x10MHz for each successful bidder anyway*”.

Aggregation risks may also exist where frequencies in different bands are complementary, e.g. because a combination of spectrum in different frequency ranges is needed for the efficient provision of coverage and capacity. Aetha notes that “[i]n respect of the existing mobile operators, different views have been expressed ranging from the valuation of the [700 and 2100 MHz] bands can be undertaken relatively independently through to valuations can only be made knowing how much spectrum the organisation has in the other bands. Essentially here there are very different views on the strengths of linkages between the 700MHz and 2100MHz band amongst different stakeholders, but it seems a large number do believe there are linkages between these bands.” By contrast, “there is no particular linkage between the 1452-1492MHz and 2100MHz bands.” Overall, Aetha concludes that “linkages across all three bands are limited – and primarily are in relation to the valuation of spectrum in each band in view of the general substitutability of capacity.”

3 Assessment of candidate auction formats

3.1 Framework for assessment

Our assessment criteria capture the Ministry's objectives for the award, set out above, and translate them into more detailed requirements for the auction design. As we will discuss, the requirements are mainly determined by the primary objective of producing an efficient assignment. This objective requires a transparent and competitive auction process in which bidders are not exposed to substitution or aggregation risks, are not hampered by unnecessary complexity and where the scope for strategic bidding is minimised. Such an auction design will also generate realistic revenues and be well aligned with the additional goals identified by the Ministry.

Efficient assignment of frequencies

An auction process is most likely to generate an efficient outcome if the bids submitted indicate the value that the bidders place on different spectrum portfolios. Competition amongst bidders will see scarce frequency resources end up in the hands of those who value them most if every bidder can express its valuations for different combinations of spectrum blocks without distortions that might arise from being exposed to risk and uncertainty over outcomes, strategic complexity, bidding mistakes arising from unduly complex processes, or strategic bidding incentives. It is important that all bids that are relevant for determining the efficient outcome are placed and that these bids closely reflect the bidders' valuation. Unfortunately, there are many reasons why bidders may not place all relevant bids or why bids may not reflect underlying valuations. Looking at each of these will provide a set of criteria for the assessment of auction formats.

To ensure efficiency when lots are substitutable, the auction format should allow bidders to express substitutability

Spectrum portfolios, and potentially individual spectrum blocks, are often substitutes: depending on relative prices, a bidder might be willing to acquire one or the other portfolio (or block). Efficiency in this case requires that frequencies are aligned in line with relative valuations. This means that at the final prices each winner prefers the lots it has won to those won by others

(i.e. there would be no gains from trade between winners).¹³ Such an outcome can only be achieved if the bids submitted by each bidder reveal information about the bidder's relative valuations across different spectrum portfolios to the auctioneer.¹⁴

The risk that a bidder might end up winning some lots that are not the preferred ones at final auction prices is called 'substitution risk'. An auction design that leads to an efficient outcome should eliminate substitution risks.

Substitution risks are perhaps most easily addressed in open multi-round processes in which bidders can respond to changing relative prices by switching their demand across different lots and combinations of lots. This requires that impediments to bidders switching their demand in response to price signals should be eliminated (or at the very least minimised).¹⁵

In sealed bid processes where switching is not possible by definition, substitution risks can be mitigated by allowing bidders to reveal their full demand profile across substitutable portfolios, through a sufficiently rich set of bids for alternative combinations of lots rather than requiring them to select a specific combination of lots or to bid for individual lots (which could lead to a wide range of combinations of lots depending on which bids become winning bids).

Unmitigated substitution risks may also result in inefficiently unsold lots, because some bidders who would be willing to acquire these lots at final prices may simply have been unable to express their willingness to do so through their bids. For example, a bidder might be happy to acquire the lots at their final prices but instead wins other lots in which other bidders would also be interested, or no lots at all, because of switching impediments or as a result of not having made all of the relevant bids.

¹³ With frequency-generic blocks, individual blocks in a category are perfect substitutes, i.e. a bidder would prefer whichever lot is the cheapest by even an infinitesimally small amount. More generally, we are concerned with imperfect substitutability where bidders attribute different values to different lots or combination of lots but are happy to acquire lower-value lots if they are cheaper by a sufficient margin than higher-value ones.

¹⁴ For the avoidance of doubt, assigning the spectrum efficiently does not require bidders to know the individual competitors' relative valuations; relative valuations are reflected in the bids submitted to the auctioneer who then adjusts prices to adjust their demand.

¹⁵ Note that switching impediments often arise when bidders would want to switch between combinations of lots rather than individual lots and are thus closely linked to aggregation risks.

To ensure efficiency when lots are complementary, the auction format should minimise aggregation risks and support non-linear prices

Complementarities between lots can expose bidders to so-called aggregation risk, i.e. the risk of ending up with an unwanted subset of the lots it bid for or overpaying for those lots having made bids in the expectation of being able to win complementary lots but failing to do so. Lots are complementary when a bidder's valuation of the combination exceeds the sum of the standalone values of the individual lots (i.e. valuations are synergistic). As noted above, stakeholders have expressed the view that complementarities are limited and therefore aggregation risks should be manageable without the need to resort to an auction formation that supports package bidding.¹⁶ Equally, there should be no need for a format that supports outcomes in which the price per block can vary with the size of the spectrum portfolio won by a bidder, and potentially across bidders.¹⁷

Reducing strategic complexity aids efficiency

A further reason why bids might provide distorted signals of individual valuations is underlying strategic complexity. Strategic complexity arises mainly from uncertainty over results and the inability of bidders to control their outcomes.

¹⁶ In auction formats that support package bidding, bids for combinations of lots are assessed in their entirety rather than on a lot-by-lot basis. Because of the all-or-nothing nature of evaluating bids, bidders are never exposed to the risk of winning an unwanted subset of lots and aggregation risks are therefore eliminated. Alternatively, aggregation risks can be mitigated in auction formats that evaluate bids on a lot-by-lot basis by mechanisms that allow bidders to withdraw provisionally winning bids. However, such withdrawal opportunities can create other problems as bids cease to be committing. Therefore, withdrawals may need to be limited or penalised. Alternatively, withdrawals may be triggered automatically, based on some pre-specified conditions (as in the case where bidders can specify a minimum requirement of lots, with any provisionally winning bids for a lower quantity becoming void when the bidder fails to win at least the pre-specified quantity). Such provisions were used, for example, in the German 4G auction of 2010 or the recent UK auction of spectrum in the 2.3 and 3.4 GHz band.

¹⁷ With complementarities between lots, bidders should ideally be able to signal the extent to which the value of larger spectrum portfolios is disproportionately larger than the value of smaller ones. This means that bidders must be able to submit package bids at levels that imply different per-lot prices for differently sized portfolios. Indeed, there may be no set of uniform per-lot prices that support an efficient outcome. To see this, consider the simple case where a bidder values two blocks at more than twice the value of a single block: at any price at which such a bidder would be happy to acquire a single block, it would prefer to acquire two blocks, as this provides a greater surplus. At any price, at which the bidder would not wish to buy two blocks, it would certainly not be interested in a single block. If the efficient outcome requires that such a bidder ends up with a single block, this could not be achieved with an auction format in which the price per block is the same regardless of how many blocks a bidder acquires, except in the case where there is a substantive risk that the bidder is overpaying for the single block. This means that auction formats that do not support average per-lot prices that may differ across different packages may fail to produce an efficient outcome, particularly in terms of leaving some lots inefficiently unsold. Where there is a risk that some bidders may be assigned fewer lots than they bid for at a price that exceeds their valuation for those subsets of lots, bidders will be discouraged from fully expressing their valuation for combinations of lots

Strategic complexity is different from the complexity of the auction rules or the auction mechanism itself. For instance, the procedural rules of a first price, sealed-bid auction of a single lot are simple: the highest bidder wins and pays the amount of its bid. However, from the point of view of a bidder, determining the right bid level is strategically complex. In order to make winning worthwhile, the bid should be below the value that the bidder attributes to the item. The lower the bid, the larger the surplus enjoyed by the bidder if she wins. At the same time, lowering the bid reduces the probability of winning. With an objective of maximising expected surplus, bidders will typically need to determine their bids not only based on their own valuations for the lots, but also taking account of their expectations about the valuations and the behaviour of other bidders. These expectations could be incorrect. As a result, determining by how much bids should be reduced below value is strategically very complex.

Both substitution and aggregation risks introduce strategic complexity. In order to limit the likelihood of ending up with unwanted combinations of lots bidders will typically need to bid form expectations about the final auction prices and outcome. For instance, a bidder who anticipates switching impediments in an SMRA may need to consider the risk of being stuck with a choice made early in the auction when deciding on which lots to bid.

Bidders may also simply face practical limitations. When there are many lots on offer and bidders might be interested in a wide range of packages it can be difficult to prepare a consistent set of bids that reflect the valuations and value difference for all potential target packages.

Some formats such as the CCA aim to reduce this complexity through an open bidding phase that helps bidders to identify what packages could realistically form part of a market-clearing outcome (package discovery process). However, bidders may still be exposed to uncertainty over outcomes and may not be able fully to express their priorities over alternative combinations or lots, especially if they face budget constraints.

This is because budget constraints impose limits on the ability of bidders to express the full range of valuations in their bids, at least for the larger portfolios they might target. Bidders then face complex decisions about how bid values should be adjusted in order to maximise their chances of winning the best combination of lots that can be acquired with their budget. Bidding full value for smaller portfolios that are well within a bidder's budget and only limiting the bids placed on larger portfolios where the value exceeds the budget means that the bids will understate the incremental value of bigger portfolios. A bidder could end up winning a small portfolio where efficiency would have required her to win a larger one, even if the larger portfolio were within budget. By contrast, reducing the amount bid on the largest portfolios and all smaller ones in order to

Strategic bidding can result in inefficient outcomes, so the auction design should discourage strategic behaviour

express true value differences can lead to the bidder failing to win anything.

Bids may not reflect underlying valuations if bidders try to game the auction through their bids rather than responding to price signals and revealing their valuation of different spectrum portfolios through truthful bids. Attempts to manipulate prices or winning outcomes – often called ‘strategic bidding’ – cover a wide range of possible behaviours.

In broad terms, strategic bidding may take the form of understating demand in order to keep prices down or overstating demand in order to drive up the prices paid by others.

Understating demand aimed at keeping prices low is a typical problem of multi-unit auctions with pay-as-bid pricing. Bidders may have an incentive to reduce their demand even if *current* prices are well below their valuation for marginal spectrum blocks if they expect that doing so leads to lower *final* prices. In this case a bidder may prefer to settle for less spectrum at a lower price over trying to win more even if the additional spectrum had value well in excess of prices. The incentive to engage in this type of behaviour is strongly linked to starting prices – the lower starting prices relative to the value of spectrum, the greater the scope for realising a substantial surplus from settling for a smaller amount of spectrum early compared with trying to win a larger amount of spectrum at a higher price *even* if the bidder expects to be able to win more spectrum at final prices that are substantially below valuation.

Even where there is no unilateral incentive to understate demand to keep down prices, bidders may benefit collectively from doing so and might try to pursue tacitly collusive strategies to bring competition to an early end. The risk of such behaviour is higher where there are outcomes that all bidders may consider to be particularly likely which then provide so-called ‘focal points’ for co-ordination.

Attempts to keep prices down by reducing demand will not only reduce revenues but can result in inefficient outcomes, as bidders hide their value for additional spectrum. Where inefficiencies arise, they tend to be associated with a more even distribution of spectrum (as bidders who acquire more lots have a greater incentive to reduce demand to freeze prices), which might partly counterbalance efficiency concerns as there could be benefits from avoiding increasingly asymmetric spectrum holding that are not fully reflected in the notion of (short-term) efficiency.

The incentive to reduce demand before prices reach incremental valuations is greater when prices are very low relative to valuations and relative to the prices at which a bidder would expect to be able to win a larger amount of spectrum, as this implies large gains from bringing an auction to an early end. Thus, making sure that prices are not too low at the start of the

auction is a very effective countermeasure. This is one of the reasons for setting reserve prices at levels that are not too far from the end prices that would be expected in a competitive auction.

Price-driving behaviour may be part of tacitly collusive strategies (i.e. as punishment for deviating from the collusive outcome) but can also be aimed at exhausting other bidders' budget in order to limit competition for specific lots. Both cases require that bidders can bid for specific lots or lot categories that they do not want to win themselves, but in which their competitors are interested. In both cases, the behaviour is aimed at keeping one's own prices down by increasing, or threatening to increase the prices paid by others and is therefore consistent with the assumption that bidders are motivated by maximising the difference between their valuation of the lots they win, and the prices they pay (surplus maximisation).

Price driving strategies may be more of an issue when bidders are not purely motivated by surplus maximisation, as the theoretical auction literature assumes, but are also concerned about the outcome they achieve *relative* to those obtained by competitors. Paying less – or certainly not more – than other winners may be more important than the absolute price level. This is especially relevant in second-price auctions, where the prices paid by winners are determined by the bids made by others for additional spectrum. In such formats, bidders may have an incentive bid for more spectrum than they wish to acquire in order to drive the price paid by competitors, though of course the risk of winning unwanted blocks acts as a corrective. One needs to acknowledge that with bidders being concerned about relative performance rather than purely maximising their own surplus, even auction formats that in theory should provide strong incentives for truthful bidding (such as the generalised Vickrey auction¹⁸) are not immune to strategic bidding.

In general terms, strategic behaviour is less likely the more competitive the auction process, as the ability of individual bidders to affect outcomes is more limited. In very competitive auctions, bidders will be less able to manipulate prices, and thus they are less likely to reduce demand or overstate their needs. Therefore, encouraging competition in the auction helps generating efficient outcomes. However, strategic behaviour is a concern where participation is limited. In such cases it is important to try to promote competition for incremental spectrum.

¹⁸ There are a number of further problems with generalised Vickrey pricing in multi-unit auctions, which have been well recognised in the academic literature (see, for example, Lawrence M. Ausubel & Paul Milgrom, 2004. "The Lovely but Lonely Vickrey Auction", Discussion Papers 03-036, Stanford Institute for Economic Policy Research) and have also been discussed extensively at the auction seminar held by the Ministry

A simple and transparent auction format helps avoiding bid mistakes

Finally, bids may fail to reflect valuations because of bidding mistakes that come from complex auction rules. Simplicity and transparency in this regard are not only additional objectives but contribute to efficiency. Complex rules that are difficult for bidders to understand increase the risk of bidding mistakes and inefficient outcomes. Bidders should be able to understand how bid decisions translate into results, so this process needs to be easy to follow and transparent. For the avoidance of doubt, this does not mean that all bid decisions are laid open to all bidders at all points, i.e. it does not require an information policy that maximises transparency. Rather, bidders should be able to retain control over their own auction results and not be exposed to unnecessary uncertainty over outcomes (especially the risk of leaving the auction empty-handed without explicitly having accepted such an outcome).

At the same time, however, pushing for simpler auction rules can have adverse impacts on efficiency. Some more complex auction rules may be needed to support fluid switching or reduce strategic complexity. If there are strong complementarities, it may be that only a combinatorial auction process can ensure efficient outcomes. Such processes are by their very nature more complex, as they involve evaluation of different combinations of bids.

Pursuing mechanical simplicity for its own sake would be counterproductive in these cases, as it would inevitably increase strategic complexity through substitution and aggregation risks. Therefore, the right balance needs to be sought.

Generating realistic revenues

We understand that the objective of the auction process generating realistic revenues is subject to achieving the primary objective of producing an efficient assignment of frequencies. Taking the efficiency objective as given, concerns about raising realistic revenues would seem to be mainly about the extent to which the auction design might prevent competition (e.g. if large lot sizes do not allow bidders to compete for incremental spectrum, or if there are material switching impediments and bidders cannot express their demand for alternative lots to those which they eventually win), or affect bidding incentives (e.g. if the auction creates incentives and provides scope for bidders to bid strategically to try keep their own prices down).

We therefore interpret the revenue objective as a requirement that the auction process should, as far as is practicable, not be susceptible to strategic bidding which could depress revenues, even if such strategic bidding were not to have efficiency implications (i.e. would not affect the overall outcome, but only prices paid). This could be an issue in this award if only the three existing MNOs participate in the auction, as in this case

the scope for competition in the auction would be limited, and depending on the auction chosen there could be possibilities for bidders to coordinate with a view to keep prices down.

At the same time, we consider that the revenue objective does not suggest using an auction format that encourages bidders to overstate their willingness to pay. The reason for this asymmetry is that discouraging strategies that are aimed at keeping prices low is good for both efficiency and revenues, whereas encouraging strategies that are aimed at driving prices up might be good for revenue, but pose risks for efficiency, as discussed above.

Similarly, we consider that the revenue objective does not justify the use of measures aimed at preventing focal points if these might prevent an efficient assignment. For instance, packaging the spectrum into lots of different sizes would prevent outcomes in which the available spectrum is shared equally amongst the bidders,¹⁹ which would rule out this possibility as a focal point for market sharing – however, such outcomes may well be efficient, and preventing their emergence therefore would jeopardise the efficiency objective for the sake of higher revenues.

Overall, this means that the objective of generating realistic revenues does not imply any further assessment criteria but should be captured in the concerns about strategic bidding to keep prices low. As we discuss below, many of these concerns can be address by setting reserve prices relatively close to the prices that one would expect in a competitive auction, as this reduces the potential gains from avoiding competition for additional spectrum.

Additional goals

As noted above, there are several additional goals for the auction design, namely that the auction design should be:

- simple, in terms of both auction rules and mechanics and bid strategy and decisions;
- transparent; and
- providing freedom of choice, which means that that bidders should be able to express their demand for spectrum on a level-playing field.

As we have discussed above, simplicity of rules and mechanics and transparency of the process are also aspects that

¹⁹ For example, packaging spectrum in a way that avoids any symmetric sharing of spectrum in the 700 MHz band would require offering one block of 2x15 MHz, one block of 2x10 MHz and one 2x5 MHz block, which means that the distribution of spectrum amongst winners is largely pre-determined. Similarly, in the 2100 MHz band blocks would have to be defined so that they cannot be combined into 2x20 MHz assignments, e.g. four blocks of 2x15 MHz.

contribute towards efficiency. It is desirable to look for the simplest format that deals effectively with the need for mitigating substitution and aggregation risks and discourages strategic bidding for efficiency reasons already.

Similarly, the goal of allowing bidders freedom of choice is, in our view, a crucial part of any auction design that promotes efficiency. We have discussed above the importance of bidders being able to express their valuations for different combinations of spectrum blocks without distortions arising from exposure to risk and uncertainty over outcomes, strategic complexity, bidding mistakes resulting from complex processes, or strategic bidding incentives. Freedom of choice would be limited if bidders were hampered in expressing their demand because of substitution and aggregation risks, or because of strategic complexity. Therefore, these concerns are well covered by the considerations of efficiency above, so this additional goal does not require additional assessment criteria.

Summary

In summary, we have identified the following requirements for a suitable auction format:

- mitigate substitution risks by allowing bidders to make bids for different spectrum portfolios or to switch across portfolios in response to price signals;
- where substantial complementarities exist, mitigate aggregation risks and potentially support non-linear prices by allowing bidders to express the underlying synergies in their bids, which effectively requires package bidding;
- reduce strategic complexity by reducing uncertainty over outcomes;
- discourage strategic bidding or limit the opportunities for bidders to gain from engaging in distorted bidding behaviour; and
- be simple and transparent in order to limit the risk of bidding mistakes (though not at the expense of meeting any of the other requirements)

Given the views expressed by stakeholders, we do not consider that substantial complementarities exist, and thus conclude that mitigating aggregation risks is not a key requirement for this award. This is also reflected in Aetha's assessment, which finds that although there is some complementarity between the 700 and 2100 MHz bands (for potential users who not currently hold spectrum below 1 GHz) and between the 700 and the 1400 MHz band (for stakeholders who do not hold 800 MHz spectrum), "*there is a very limited linkage across all three bands*

(700MHz, 1452-1492MHz and 2100MHz) which mainly relates to the substitutability of capacity.”²⁰

3.2 Identifying suitable candidate auction formats

When looking at candidate formats, it is helpful to consider several broad choices in sequence, namely between:

- single-round and open multi-round processes;
- combinatorial and non-combinatorial formats; and
- pay-as-bid or second-price formats (where relevant).

These choices define a decision tree that allows us to narrow down the formats we need to consider further.

Single round vs. open multi-round processes

Open process allowing bidders to respond to price signals are better suited to meet the requirements identified above

We consider that an open, multi-round process is better suited to achieving the requirements identified above than single-round sealed formats.

Given the many lots available for this award, which are likely to be good substitutes but possibly also complementary to some degree (both in terms of synergistic valuations of spectrum within a band and across portfolios), the only reasonable option for running a sealed bid process would be a combinatorial sealed bid. Even if complementarities are weak, bidders would need to be able to bid on mutually exclusive packages in order to be able to express their preferences for different spectrum portfolios.

Under this format we would rely heavily on bidders expressing a sufficiently rich range of preferences over spectrum portfolios that they believe might form part of a market-clearing outcome in order to produce an efficient assignment.

However, there are several reasons why the bids received may fail to provide the necessary information to identify an efficient outcome:

- Making a large set of bids for all relevant packages without the package discovery functionality offered by a multi-round process is certainly challenging, and mistakes can easily be made.
- Bidders are furthermore exposed to substantial uncertainty over what they will win. In practical terms, this can create substantial governance issues, as bid teams would have to seek authorisation to pursue several targets with little

²⁰ Aetha report, p. 54.

information about which of these they are likely to win, let alone any control over which one they will win.

- Managing budget constraints would be extremely challenging, as discussed above, as expressing valuation differences across portfolios is very difficult without any knowledge about the largest package that the bidder might realistically win. Any attempts to address this uncertainty by submitting bids for only a limited range of packages will jeopardise the chances of generating an efficient outcome.
- If a pay-as-bid pricing rule were to be used, this would create substantial challenges in terms of identifying optimal bid amounts. Trading off higher winning probabilities against lower surplus in case of winning is a formidable task even when bidding for a single lot. Finding optimal bid amounts for a large number of packages will make the task disproportionately more difficult, in particular if there has not been any information disclosed that will assist bidders in identifying their optimal bids amounts (as would be the case if the sealed bid were preceded by some open bidding rounds).
- If a second price rule were to be used instead, this would remove the problems of identifying the optimal amount of bid shading but would leave the strategic complexity of dealing with budget constraints in terms of needing to express value differences between spectrum portfolios. In addition, incentives for placing price-driving bids if bidders were (also) concerned about relative performance could undermine the efficiency of the process. The substantive uncertainty over outcomes makes such behaviour perhaps riskier but at the same time it becomes easier to make mistakes.

A sealed bid process, though simple in terms of the rules (especially with a pay-as-bid rule) is not transparent and leaves bidders with very limited control over their outcome.

By contrast, open formats give bidders the opportunity to respond to changing prices that signal relative scarcity. Bidders may not only use these price signals to confirm or update their own valuations helping them to deal with common value uncertainty, but more importantly will gain a better understanding of the outcomes that are likely to emerge.²¹ This allows them to discard target portfolios that become unaffordable given their budgets and narrow down the range of combinations of lots they can expect to win.

²¹ To the extent that bidders' valuations are driven by common but uncertain factors, allowing bidders to observe each other's behaviour efficient outcomes are more likely in auction processes that help to mitigate such common value uncertainty. This is typically an argument used to support the use of open multi-round processes, potentially with the maximum amount of information about each bidder's bidding behaviour being made available at each point, as being able to observe each other's behaviour could help bidders with updating their own valuations.

Given the considerations above, open multi-round formats are likely to perform better than a single-round sealed bid process for this award.

Combinatorial vs. non-combinatorial formats

Having ruled out single-round sealed bid processes, we can now consider the choice between combinatorial and non-combinatorial formats. A detailed description of these formats is provided in Annex A.

- Open multi-round combinatorial formats include the Combinatorial Clock Auction (CCA) and its Enhanced CCA variant (ECCA) as well as the CMRA, the simple clock auction (SCA) and – for sake of completeness – the Simultaneous Multi-Round Ascending Auction with hierarchical package bidding (SMRA-HPB). As explained in more detail in the Annex, we consider the SMRA-HPB to be viable only in the case where it is possible to define non-overlapping packages at different hierarchy levels that closely match the preferences of different bidders.
- Open multi-round non-combinatorial formats include the standard SMRA, variants offering augmented switching opportunities through greater opportunities for withdrawal of standing high bids (SMRA-AS), the so-called Clock Plus auction (CA+) and the SMRA-Clock Hybrid (or clock auction with provisional winners).

As we have discussed above, without strong complementarities there is no need for using the more complex combinatorial formats: whilst with strong complementarities it may be difficult, if not impossible, to achieve an efficient outcome without support of package bidding and without permitting non-linear prices (i.e. package prices that cannot be decomposed into unique per-lot prices)²², the additional complexity of such designs must be justified by the risk that an efficient outcome may not be achieved without a combinatorial format.

As noted above, we understand that the views expressed by stakeholders indicate that complementarities are not strong and that prospective bidders consider that aggregation risks are

²² Therefore, we would not recommend the SCA if synergies are likely to be material, which would then leave us with the CCA, ECCA and CMRA. A variant of the clock auction – the so-called clock auction with clinching – permits non-linear prices by assigning lots at the prevailing clock prices as soon as they are no longer contested (i.e. aggregate demand from other bidders is less than supply). This essentially introduces opportunity-cost based pricing per lot in a simple manner and removes incentives for demand reduction to keep down prices (see Ausubel, L (2004) “An Efficient Ascending-Bid Auction for Multiple Objects,” American Economic Review 94(5)). However, this format works well only with a single lot category as with multiple lot categories and the ability to switch between it is far from clear what lots should be clinched at what price.

manageable without support of package bidding. This suggests the choice of a non-combinatorial format.

Pay-as-bid or second-price formats

Non-combinatorial open multi-round auctions such as the standard SMRA, the SMRA-AS, the CA+ or the SMRA-Clock Hybrid use a pay-as-bid rule (though some variants of this rule might be employed, for instance requiring that all winners of lots of a given type pay the lowest winning or the highest losing bid for that lot type). Thus, if a non-combinatorial format is chosen, there is no need to consider the pricing rule.

If a combinatorial format had to be used, there would be a choice between formats that use a pay-as-bid rule (such as the CMRA and the SCA) and those that employ a second pricing approach (such as the CCA, which sets prices on the basis of opportunity costs calculated from the bids made by bidders, and the ECCA, which sets prices with reference to the largest bids that competitors could make under the activity rules). Given the simplicity of pay-as-bid pricing²³ and the potential concerns about the impact of the asymmetry in the amount of spectrum that different bidders can acquire under the caps on bidding

²³ For example, in a single-unit open auction, a bidder can simply carry on as long as its valuation exceeds the current bid level. Bidding stops when the second highest bidder reaches its valuation, the lot is won by the bidder with the highest valuation, and the outcome is efficient. This no longer holds in multi-unit auctions, as there is an incentive for bidders to reduce demand to keep prices low.

behaviour, we would prefer a pay-as-bid format over a format that relies on opportunity-cost based pricing.²⁴

As discussed above, under a pay-as-bid rule, there is a risk that the auction process might not produce realistic revenues, even if it were to result in an efficient outcome. However, we consider that this risk can be addressed through setting appropriate reserve prices and does not require giving up the benefits of simplicity and transparency that are associated with successful bidders paying the amount of their bids.

Summary

Figure 1 below summarises these considerations and identifies the candidate auction formats for further consideration.

We focus on open formats. The first choice then is whether to use a combinatorial format, which would be justified if we expect there to be strong complementarities across lots.

If a combinatorial auction is not required, then we are left with different variants of the SMRA and hybrid models combining elements of the SMRA and the clock auction format. All these formats use a pay-as-bid rule and are therefore susceptible to strategic demand reduction. However, we consider that the risk that this implies for achieving realistic revenues is best addressed by setting appropriate reserve prices rather than by switching to a combinatorial format that could employ a second price rule, or by using a sealed bid process with the associated risk of creating an inefficient outcome. Setting realistic reserve prices is, in our view, a more appropriate response than

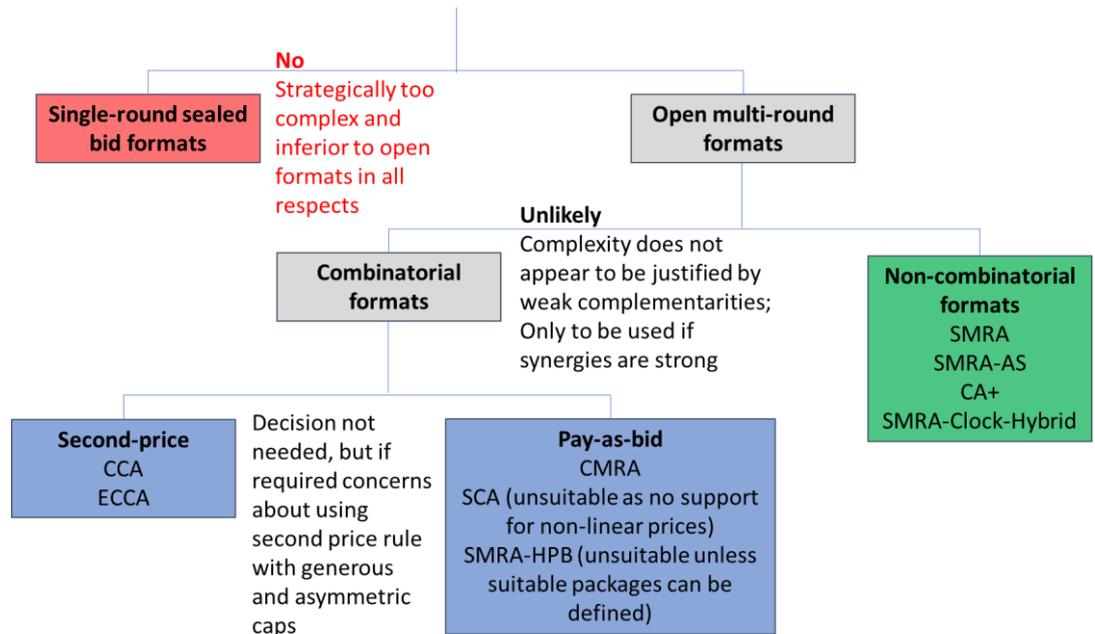
²⁴ Using a pay-as-bid rule involves substantial strategic complexity in processes with a sealed bid component, as bidders need to determine their optimal bid amounts trading off the lower probability of winning from bidding less against paying a lower price if they win. This is why the CCA (which always includes a sealed bid component in the form of the supplementary round) and the ECCA (which may skip the supplementary round but in any case uses the bids that bidders could hypothetically have placed in order to determine prices) use a variant of the second-price rule, which requires that no bidder or combination of bidders would be prepared to pay more for the lots obtained by winners than the winners pay (i.e. that prices are in the core). The modified second-price rule should make it easier for bidders to determine their bid amounts as these mainly determine the probability of winning, whilst the price is (largely) determined by the bids from losing bidders. However, as prices paid are divorced from the bids placed, such a pricing rule creates uncertainty over a bidder's financial liability and makes it more difficult to manage budget constraints.

Where bidders are strongly motivated by relative performance, they may also be concerned about placing bids that ensure that others pay sufficiently high prices for their winnings. In this respect, using a second price rule is potentially more of a concern where spectrum caps have an asymmetric impact on bidders' ability to bid for additional spectrum in the auction. This is the case under the spectrum caps proposed for the auction. Under such asymmetric constraints the ability of bidders to set each other's prices is uneven and attempts to exploit this asymmetry through strategic bidding may result in inefficient outcomes.

choosing a more complex auction format if this is not required for efficiency reasons or using an auction format that creates maximum uncertainty for bidders.

Should a combinatorial format be required, we would be left with a choice between the CCA/ECCA and the CMRA. The SMRA-HPB format incorporates some package bidding in a simpler SMRA framework; however, its use in practice is limited to cases where it is possible to define non-overlapping packages that match the preferences of bidders.

Figure 1: identifying suitable candidate formats



3.3 Recommended auction format

The views expressed by stakeholders suggest that complementarities across bands are limited and that synergistic valuations within bands should not be so strong as to require package bidding. Bidders should be capable of dealing with the moderate aggregation risks in simpler non-combinatorial formats. Therefore, we do not consider that a combinatorial format is required for this specific award.

Within the non-combinatorial formats, we would recommend using the SMRA-Clock-Hybrid format. The reasons for this are as follows.

The well-tried and tested standard SMRA format provides a reasonable starting point. However, the standard version can be procedurally very inefficient with many identical lots, taking a very long time to resolve. Many rounds may be required for prices to increase for several substitutable lots when there is little excess demand overall.

These shortcomings would not be addressed by the augmented switching variant of the SMRA, which has typically been used to mitigate the risk of fragmented outcomes when spectrum is offered in the form of frequency-specific blocks. At the same time, the format introduces further complexity which is unjustified when offering the spectrum as frequency-generic lots. There is little need for switching bids across generic blocks and the rules on re-activation of earlier bids create further risks for bidders.²⁵

Using a clock mechanism for collecting bids within an SMRA framework has substantial advantages. Grouping lots within categories with the same price tag allows us to apply price increments more rapidly and uniformly across substitutable lots. Therefore, the SMRA-Clock Hybrid has strong procedural advantages compared with the standard SMRA, especially where there are multiple frequency-generic lots on offer.

Being a variant of the clock auction, the CA+ would also achieve this. However, the CA+ creates standing high bids implicitly rather than explicitly, and these are only communicated to bidders once they would become effective in preventing switching or demand reductions because permitting such bids would leave excess supply in a lot category. The CA+ then denies part of such switches or demand reductions and thus exposes the bidder to the risk that some of its switches/reductions are accepted without being able to revert to its earlier demand.

By comparison, the SMRA-Clock Hybrid is more transparent and has simpler rules. By identifying standing high bids after every round, it gives bidders certainty over whether and to what extent they can switch or reduce demand and refrain from switching where this could expose them to the risk of ending up unwanted combinations of lots. This contributes to the predictability of the SMRA auction mechanism. The determination of standing high bids allows bidders to keep track of their position while the outcome slowly cements: at any point, bidders are able to calculate the price they would need to pay for the lots on which they bid and likely price increments in the following few rounds. Bidders can then progressively adjust their demand to settle for the lots they can realistically acquire. Using standing high bids makes it easy for bidders to bid progressively in response to being outbid.

A further advantage of the SMRA over the CA+ is that with more than two lot categories bidding mechanics are much simpler. This is because bidders who switch from one lot category into more than one other category in a CA+ will need to specify their switch preference for the case where the switch

²⁵ If the option to withdraw standing high bids were needed to facilitate switching, we would instead suggest tighter conditions on withdrawals without reactivation of previous standing high bids, but overall consider this not to be necessary.

is partially denied. For example, a bidder wishing to switch from bidding on 2100 MHz spectrum to a combination of 700 MHz and 1400 MHz blocks would have to specify whether it places greater weight on switching into the 700 MHz band or into the 1400 MHz band in the case that part of the switch were denied. Such switch preferences are unnecessary in the SMRA-Clock Hybrid format, as the bidder knows the extent to which it can switch demand.

Employing a simple pay-as-bid rule, the SMRA-Clock Hybrid format is transparent and easy to understand. All pay-as-bid formats in multi-unit auctions are of course susceptible to bidders reducing their demand to keep prices low. However, as incentives for engaging in strategic demand reduction are stronger for bidders who are expecting to acquire a greater number of frequency blocks, this would tend to result in a more even distribution of spectrum than might otherwise be the case. In any case, the incentives for engaging in such behaviour (and any negative revenue impact) should be mitigated by setting appropriate reserve prices.

For the avoidance of doubt, we note that very low reserve prices may increase the risk of failing to achieve a realistic revenue and could potentially also result in inefficient outcomes. However, even in the face of these risks we would not recommend the use of a model with a second price rule, owing to the asymmetric constraints on bidders or of a sealed bid format, owing to the risk of inefficient outcomes.

An additional advantage of the SMRA-Clock Hybrid is that it would deal somewhat better with any residual aggregation risks than the standard SMRA. Unlike in the standard SMRA, we can ensure that in each lot category there is at most one bidder whose demand is only partially satisfied in any given round, through first ranking bidders and then satisfying the demand expressed by bidders in turn. This reduces any remaining aggregation risks in relation to the total bandwidth demanded by a bidder in a given lot category.

Finally, we consider that the SMRA-clock Hybrid works well with the spectrum caps adopted for the award. In particular:

- the spectrum caps allow for a wide range of outcomes, which justify the use of an open format that mitigates uncertainty about the final auction outcome;
- whilst the caps still allow bidders to bid for large portfolios that include lots in which they are not ultimately interested, the SMRA-Clock Hybrid format discourages such behaviour through the designation of standing high bids, as these create a real risk that a bidder could win any lots on which it bids;
- the use of a pay-as-bid rule neutralises the potential price asymmetries that could arise as a result of the asymmetric bidding restrictions (flowing from the application of equal caps with uneven existing spectrum holdings) under a

second price rule, where different effective limits on the amount of spectrum on which bidders can bid means that they can impose different opportunity costs on their competitors.

We set out specific design elements of the proposed format in the next section, and provide a set of draft auction rules in Annex B.

Table 3 below summarises our assessment of the various auction formats in the specific context of this award in a simple scoring table against the Ministry's objectives. In interpreting this table, it is however important to realise that this presentation does not capture the contingencies and the trade-offs discussed above. A more detailed explanation of the reason for these scores can be found in Annex A.6.

As a final point for discussion, we consider what our recommendation would be if synergies were sufficiently strong to justify the use of a combinatorial auction format. In this case, we would have concerns about the CCA/ECCA owing to its use of a second-price rule, given that the effective spectrum caps in the auction are asymmetric. Though the second-price rule removes the incentives for strategic demand reduction and thus may be considered to be more conducive to achieving realistic revenues, the magnitude of opportunity costs that are reflected in final prices is linked to the extent to which bidders can compete for additional spectrum and therefore, in total, to the amount of spectrum on which bidders could place bids under their respective caps relative to the total spectrum available.

Amongst the other formats, the SCA seems inappropriate if synergies are strong, as the elimination of aggregation risks comes at the cost of a non-negligible risk of inefficiently unsold lots owing to the pricing uniformity imposed by the format. Overall, this would suggest the use of the CMRA, which provides greater certainty over outcomes and prices. Given the concerns raised by stakeholders about the use of the CMRA, this implies that there would need to be very strong underlying complementarities for a combinatorial format to be justified.

Table 3: Comparison of auction models

Objective	Sealed bid	CCA/ECCA	CMRA	Simple clock	SMRA	SMRA-Clock-Hybrid	Clock+
Efficiency*	✗ Risk of missing bids and mistaken expectations	✓✓ Supports fluid switching between portfolios	✓✓ Supports fluid switching between portfolios	✓ Supports fluid switching between portfolios, but has risk of unsold lots unless accompanied by provisions for making exit bids	✓ Supports switching between portfolios but with some impediments	✓✓ Supports switching between portfolios minimising switching impediments	✓ Supports switching between portfolios but with some impediments that will manifest as incomplete switches
Realistic revenue**	✓ Limited incentives for strategic demand reduction	✓ No incentive for strategic demand reduction, but reliant on bidders being willing and able to express value for additional spectrum	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices
Simplicity***	✗ Rules are simple, but decision making is strategically complex	✗ Complex rules and uncertainty over final liability (especially in the CCA) makes managing budget constraints difficult	✗ Complex rules, potentially complex bid decisions	✓✓ Simple rules and simple bid decisions	✓✓ Simple rules and simple bid decisions	✓✓ Simple rules and simple bid decisions	✗ Complex rules and uncertainty over what parts of a bid will be accepted
Transparency	✗ Substantial uncertainty over outcomes and no visibility of other bidders' decisions	✗ Uncertainty over outcomes especially in the CCA; calculation of discounts in ECCA not transparent	✓ No uncertainty over exposure and full control of own outcome, but process of establishing price increments may not be fully transparent	✓✓ No uncertainty over exposure and full control of own outcome	✓✓ No uncertainty over exposure and full control of own outcome	✓✓ No uncertainty over exposure and full control of own outcome	✓ No uncertainty over exposure, but potential difference between bids places and processed bid
Freedom of choice*	✓ (✗) Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids without indication of winnable portfolios	✓ Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids	✓ Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids	✓ Bidders can express demand for preferred portfolio at round prices	✓ Bidders can express demand for portfolios, potentially limited by standing high bids	✓ Bidders can express demand for portfolios, potentially limited by standing high bids	✓ Bidders can express demand for portfolios, but bids may not be accepted fully

* Assuming that complementarities between lots are limited and that there are no material aggregation risks.

** Assuming reserve prices are set close to prices expected in a competitive auction.

*** Both mechanical and strategic.

✗ - does not achieve objective; ✓ - performs well in achieving objective; ✓✓ - performs very well in achieving objective.

4 Proposals for detailed auction provisions

4.1 Lot categories, standing high bids and price increments

We propose to use a single lot category for each band, with the number of lots on offer being determined by the available spectrum and the individual block size.

If spectrum will be offered in the form of 2x5 MHz blocks in the 700 and 2100 MHz bands, and in the form of 5 MHz blocks in the 1400 MHz band, there would be the following lot categories:

Table 4: Proposed lot categories

Lot Category	Number of lots	Size
700 MHz FDD	6	2x5 MHz
1400 MHz SDL	8	5 MHz
2100 MHz FDD	12	2x5 MHz

Establishing the need for price increases

When using generic lot categories, the rule that new bids on a lot must exceed the standing high bid on a lot must be adapted to reflect the fact that we do not set prices for individual lots but for whole categories. In this context, a new price level will be required for a lot category if demand at the current price level exceeds or exactly matches supply, as the price will only need to increase to invite higher bids when all standing high bids are at the same price level. Conversely, if the demand at the current price is less than the number of lots available, the price will remain unchanged even if we have standing high bids on all lots (some of which will in this case be at a lower price), in order to allow for new bids at the current price to outbid earlier standing high bids at the lower price. Whenever the price remains unchanged, standing high bids would take precedence, as they may only be outbid by new bids at a higher price. An example of this is provided in Box 1.

Box 1: Price increments when using generic lot categories

Suppose there are two lots available in a category and the price in round 1 is 10. Suppose we receive a bid for one lot from each of Bidder A, B, C and D.

We select two of these bids as the standing high bids at random (which has the same effect as picking standing high bidders for individual lots at random in a standard SMRA when multiple bids are received at the same level). Suppose we select the bids from bidders A and B. There is excess demand at this price, so the price must increase in the following round, say to 11.

A and B hold standing high bids and therefore do not need to bid at this price (though they would be free to do so). Assume that C and D make bids at the price of 11. These bids are at a higher price than the previous standing high bids, and thus become the new standing high bids. Demand at a price of 11 exactly meets supply, and we need to increase the price in order to invite new bids at a price that exceeds the standing high bids, say to 12.

Assume that A is not prepared to pay this amount and leaves the auction (i.e. A does not place a further bid). B, by contrast, submits a bid at 12. The bid from B will outbid one of the current standing high bids – suppose that it is C's bid that will be displaced. The other standing high bid will remain unchallenged at this point.

Therefore, we now have standing high bids at two different levels. A new bid at a price of 12 would still outbid the standing high bid from D at a price of 11. As demand at round prices is less than supply, we do not increase the price.

In round 4 therefore the price remains at 12. Suppose that bidder C bids at the new price level. This bid will replace the standing high bid from D (note that D could have increased its standing high bid by bidding in this round at well, in which case we would have ranked both C and D at random). Now both standing high bids are at 12, and hence we must increase the price to allow for higher bids to outbid the current standing high bids. D can now bid back for a lot, and so forth.

Overall, we have thus the following bidding process:

Round	Price	Bids received from	standing high bids (bidder@amount)	Aggregate demand at round price
1	10	A, B, C, D	A@10, B@10	4
2	11	C, D	C@11, D@11	2
3	12	B	B@12, D@11	1
4	12	C	B@12, C@12	2
5	13

Determination of standing high bids

In the traditional SMRA bids for each lot are evaluated independently. The same could be achieved in this case if individual bids were ranked at random and declared to be standing high bids in this order until all lots in a category have a provisional winner. However, this means that there may be multiple bidders seeking multiple lots who have only part of their demand designated as standing high bids and who may

therefore face some risks when trying to switch across lot categories.²⁶

Therefore, we propose an alternative approach that considers the full demand expressed by each bidder when evaluating bids on the lots in each category. This mitigates bidders' exposure to any residual aggregation risks (however small) and facilitates switching across categories.

Specifically, we will establish standing high bids in each lot category by ranking bidders in the category rather than their individual bids at random (where rankings can differ across lot categories). We then provisionally assign all lots in a given category bidder by bidder, starting with the highest ranked bidder and satisfying the demand for each bidder in descending order of rank until the supply of lots is exhausted. This ensures that at most one bidder will have only part of its bid provisionally accepted. All other bidders have either their entire demand designated as standing high bids, or do not hold any standing high bids in that category.

This is a moderate departure from the standard SMRA auction rules. The process is described in Box 2.

Box 2: Selecting standing high bids by ranking bidders in each category

Suppose that we receive bids from three bidders (A, B and C) for blocks in the 700 MHz band where six lots are available. Suppose that A bids for four lots, B for three lots and C for two lots.

We first rank bidders A, B and C at random. Suppose we obtain the ranking B, A, C. We then assign standing high bids by considering each bidder sequentially in the order of this ranking.

- We start with B: We have six lots available (as we have not yet provisionally assigned any lots). B has bid for three lots. We can provisionally assign to B all the lots it has bid for, so B becomes the standing high bidder on three lots.
- We continue with A: We have now three lots remaining. We can provisionally assign to A three of its four lots, so A becomes the standing high bidder on three lots.
- We have no more lots available, so we stop considering any other bidders.

At the end of this process we have the following standing high bidders: B on three lots; A on three lots. C is not a standing high bidder.

This method ensures that we will accept at most one partial bid in each lot category (in this example, from A), while for all other bidders we accept either the whole bid or nothing at all in that category.

We note that under this approach a bidder who has only part of its demand designated as standing high bids will gain some

²⁶ Even where the value of additional lots might be decreasing with the number of lots, the value of additional lots is still likely to be greater if contiguous to other lots. Therefore, even if aggregation risks might not be an issue with respect to bidders not being able to obtain all the bids they bid for, they may still have an interest in obtaining multiple lots in the same band, rather than being left with fragmented holdings across bands.

information about the total demand from higher ranked bidders, which in combination with spectrum caps may reveal some information about the number of other bidders competing. However, we believe that this small informational advantage, which would be enjoyed by a random bidder each round, is justified by trying to keep at most one bidder with only partly satisfied demand.

We also note that other ranking criteria could be employed – e.g. ranking bidders in ascending order of the size of their demand in order to maximise the number of bidders who have their entire demand designated as standing high bids. Another option would be to establish first whether it is possible to fill the given supply by accepting only whole bids and use ranking only if this is not feasible. This would be feasible in the example above by accepting the bid from bidder A (four lots) and bidder C (two lots) as standing high bids. However, this would also give preferential treatment to specific bidders that provide a better fit, as these bidders would be more likely to become standing high bidders.

More generally, any approach that involves prioritising bidders in some way rather than simply ordering them at random could be criticised for being discriminatory and is likely to create systematic informational advantages for specific bidders. Therefore, we propose not to use any such approach.

The approach to establishing standing high bidders would become more complex for subsequent rounds if bidders were able to retain some standing high bids at a given price level alongside submitting bids for additional lots at a higher price level. To address this, ***we propose to require that all active bids from a bidder in a given category must be at the same price level***; this would require that bidders who wish to submit any bids at a new price level must also raise any standing high bids they may hold at the previous price level in that category.

This restriction would speed up the process by contributing to a steadier increase of prices, though it will remove some benefits for bidders who might otherwise be able to retain some of their standing high bids at a lower price level. However, we consider that these benefits are likely to be small, especially if small price increments are used once there is little excess demand. If the requirement that bidders with partial standing high bids must increase their existing bids were to be considered problematic, it would always be possible to set final prices for all bidders at the level of the lowest standing high bid in a lot category.

A further question is how existing standing high bids should be displaced where the number of new bids received is smaller than the number of lots, and where new bids may be received at the level of existing standing high bids. For this, we propose to use a simple queue system where new bids received are added to the queue, displacing an equal number of standing high bids at the end of the queue. If we retain the ranking of

bidders that has been used to establish standing high bids in the first instance, this also ensures that bidders who hold partial standing high bids will be outbid first. An example is shown below.

Box 3: Selecting standing high bids when new bids are received

Starting from the previous example (Box 2), suppose that the initial bids have been placed at a price of 10, and that standing high bidders have been selected as above, so we have the following situation in the 700 MHz band (note that the individual 'slots' do not correspond to specific frequency blocks, but only indicate places in the queue; partial standing high bidders are shown in italics).

B@10	B@10	B@10	<i>A@10</i>	<i>A@10</i>	<i>A@10</i>
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As all standing high bids are at the current price level, the price needs to increase – say to 11. Assume that B does not raise his standing high bids, but that A bids back on four blocks, and C bids on 2 blocks. Suppose that our random ranking of A and C places C in first place. This means we get a ranking of bids at the new price that looks as follows:

C@11	C@11	A@11	<i>A@11</i>	<i>A@11</i>	<i>A@11</i>
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Pushing these standing high bids in at the front of the queue means that all current standing high bids are displaced. As all standing high bids are at the current price level, the round price increases to 12.

Suppose in the next round we only receive bids from B. All these bids become standing high bids, and pushing them in at the front of the queue produces the following result:

B@12	B@12	B@12	C@11	C@11	<i>A@11</i>
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As not all standing high bids are at the current price level, the price does not need to increase but stays at 12. Now suppose that A bids back on four blocks at this price. Pushing those bids in at the front of the queue then produces the following result:

A@12	A@12	A@12	A@12	B@12	B@12
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Now all standing high bids are at the current round price and a further price increase is necessary.

Although this approach gives preference to new bids received over existing standing high bids even if the price has not changed, we believe that the impact of this is negligible if there are small price increments and excess demand is limited.

4.2 Activity rules

To ensure that bidding is progressive open, multi-round spectrum auctions typically use an activity rule that prevents bidders from increasing their demand when prices increase. Bidders withholding demand until late in the auction in order not to disclose information to their competitors would undermine the benefits from using an open, multi-round format.

It is possible to run the auction without an activity rule, allowing bidders freely to select the lots for which they wish to bid in each round and increasing their demand relative to the

preceding round if they wish to do so. However, without an activity rule the auction could take a long time to resolve if some bidders simply wait for demand to fall away before making their own bids. Not using an activity rule may also facilitate bid strategies that do not reflect bidders' demand, but that are aimed at distorting the auction outcome relative to that which would result from a competitive process where bidders bid according to their demand at prevailing prices.²⁷ Therefore, we would not recommend running the auction without an activity rule.

Measuring demand as total bandwidth in MHz

A very simple activity rule would be to require that the total demand from each bidder, expressed in MHz, must be non-increasing as the auction progresses: as prices may only remain unchanged or increase from one round to another, bidders should not demand more bandwidth. The activity rule would simply stipulate that the total bandwidth of lots for which a bidder is bidding in a given round cannot exceed bandwidth for which it was bidding in the preceding round.²⁸

Eligibility points

When lots are heterogeneous, simply adding up bandwidth across different bands may not be appropriate, and therefore another measure to express a bidder's total demand is needed. Typically, this measure is provided by eligibility points associated with individual lots, and the requirement that the activity of a bidder, measured as the sum of eligibility points for all lots on which the bidder is bidding, must not increase as the auction progresses. This activity rule is implemented by establishing an 'eligibility' level for each bidder in each round, which establishes the maximum level of activity allowed for the bidder in that round, and which is adjusted round on round by setting it to the bidder's activity in the preceding round.

Using eligibility points provides the flexibility of using different weights for different lots when measuring demand.²⁹ At the same time, the eligibility points per lot will establish a permitted ratio of two-way substitution between different lots (under a strict activity rule that establishes that activity cannot increase).

²⁷ For instance, bidders could try different demand reduction strategies to achieve a tacitly collusive outcome without restricting their own ability to compete for more spectrum if other bidders do not play along. Similarly, not using an activity rule could facilitate predatory or punishment strategies where a bidder could drive the price of the lots desired by competitors in order to bring them close to their budget or threaten them with high costs if they do not agree to a tacitly collusive outcome.

²⁸ In the case of an auction which determines standing high bidders, such as the SMRA and some SMRA-clock hybrids, the bids of a bidder in a given round include both the standing high bids from previous rounds that the bidder maintains and the new bids placed in the round.

²⁹ Notice that if we assign eligibility points in proportion to the bandwidth of lots, this rule is equivalent to requiring that the total bandwidth bid for cannot increase, discussed earlier.

Switches that involve a reduction in activity will be irreversible.³⁰ Similarly, once a bidder has reduced its activity below the level of eligibility points of larger lots it will be unable to switch from smaller lots to larger lots, even if larger lots become relatively more attractive following an increase in the price of smaller lots. This may lead to outcomes in which competition eventually focuses only on smaller lots (as bidders still competing are unable to bid for larger lots), which might then end up selling at a relatively high price.

Therefore, the options for switching demand under a rule that strictly prevents bidders from increasing their activity depend crucially on assigning the appropriate number of eligibility points to different lots.

For the upcoming award of spectrum in the 700, 1400 and 2100 MHz bands, we understand that substitutability across different bands is expected primarily to relate to all bands being regarded as providing capacity.³¹ Therefore, we would recommend that eligibility points be assigned in proportion to the capacity provided by each lot, i.e. its bandwidth, as this is most conducive to switching across different portfolios that provide similar capacity.³²

Such switching in response to price differences should be expected to lead to prices reflecting any residual value differences between the bands. Linking eligibility points to an ex-ante estimate of such value differences by contrast could potentially impede switching.

It is also worth pointing out that the risk of 'parking' eligibility on cheap lots, which could distort price signals, is limited by the fact that standing high bids cannot be withdrawn. However, we

³⁰ A bidder switching can switch to a portfolio with fewer eligibility points but will not be able to switch back to the previous package (or switch to any packages with greater eligibility) as it will not have eligibility to do so.

³¹ Aetha, *op cit*, p 53.

³² A possible complication arises with respect to substitutability between FDD and SDL spectrum, where it is unclear whether we should take the total bandwidth of lots or rather focus on the on the downlink bandwidth provided by lots, as capacity constraints are more likely to arise in relation to downlink. If bidders are likely to substitute on the basis of the total bandwidth of lots, then it would be appropriate to assign eligibility points in proportion to the bandwidth of each lot (so 5 MHz of SDL spectrum would have half as many eligibility points as 2x5 MHz of FDD spectrum). However, if substitution between SDL lots and other lots is based on the downlink capacity, then it would be appropriate to assign eligibility points in proportion to the downlink bandwidth of each lot (in which case 5 MHz of SDL spectrum would have the same eligibility points as 2x5 MHz of FDD spectrum). We suggest that eligibility points be assigned in proportion to total bandwidth unless stakeholders in their responses to the consultation of the auction documentation make a strong case for assigning eligibility points in proportion to downlink bandwidth only.

Note that

note that setting eligibility points that reflect estimated proportional value differences may be appropriate in other cases, for instance if the main reason for switching arises from the need to stay within an overall budget (in which case bidders might wish to expand bandwidth when switching to cheaper lots and contract it when switching to expensive lots), rather than trying to achieve a desired level of bandwidth in the band that offers the best value/price deal.

On balance, we suggest that the Ministry assigns eligibility points in proportion to the total bandwidth of each lot, so that, for example, a 2x5 MHz block of FDD spectrum would have twice the number of eligibility points associated with a 5 MHz SDL block.

Relaxation of the activity rule

It may be possible to relax the activity rule somewhat in order to mitigate switching impediments from the application of eligibility points when there are heterogeneous lots. One option is to use a looser requirement that allows bidders to go slightly over their activity in the preceding round, which would facilitate switching alternative lots combinations that have similar but not exactly the same eligibility points but without allowing bidders freely to increase their demand to any level.³³

The level of flexibility given to bidders depends on the activity requirement. The key question then is to determine what a reasonable activity requirement is. The looser the activity requirement, the greater the flexibility for switching, but this also allows bidders to withhold their demand until late in the auction or to switch strategically to try to distort the outcome of the auction.

For the upcoming award we do not consider that there is a strong case for using a variable or flexible activity requirement:

- we propose to use lots that are of similar size (in terms of bandwidth and eligibility), in which case there is no risk of switching impediments between lots of different size;
- whilst it is possible that there could be some switching impediments for bidders considering switching between different bands, in practice we observe that bidders often seek a portfolio of spectrum including all available bands. It is therefore less likely that bidders seeking larger amounts of bandwidth would consider completely switching their

³³ Such a rule could, for example, stipulate that the bidder's eligibility will only be adjusted if the bidder's activity falls under a predefined proportion (the 'activity requirement') of its eligibility; and that in this case the bidder's eligibility is not set to the bidder's activity in the preceding round, but to the maximum eligibility level under which the bidder's activity in the preceding round would not have triggered an adjustment of eligibility. Thus, if the activity requirement is set to X%, then if the bidder's activity in the round is below X% of its eligibility, then its eligibility level for the following round will be set to its activity level in the present round divided by X%, which is to allow the bidder to preserve the highest new eligibility level that can be sustained with its current activity.

demand across bands. Rather they might consider switching demand for incremental spectrum.

Therefore, we recommend that the activity requirement be set at 100% throughout the auction.

Revealed preference

For the sake of completeness, it is worth mentioning the possibility of using a revealed-preference approach to relaxing activity rules (like that used in some CCAs and the CMRA). Under this approach, a standard activity rule applies by default, but a bidder can make bids for a package with an activity that exceeds its eligibility for the round if this is consistent with the preferences that the bidder has expressed through previous choices.

Box 4: Revealed preference

Let p_r^X denote the price of package X in round r , and V^X denote the value that a bidder places on package X .

Suppose that in a given round n , the bidder has eligibility to bid for package A , but bids for package B instead. If the bidder has bid in accordance with its preference, this choice reveals that at the prices in round n , the bidder prefers B to A . If we assume that the bidder prefers the package that maximises its surplus, measured as the value it places on the package minus the price for the package, then the revealed preference indicates that $V^B - p_n^B \geq V^A - p_n^A$.

Conceptually, suppose that a bidder is considering two different packages (A and B) with different eligibility (such that the total eligibility of lots in A is greater than the total eligibility points of lots in B). Now suppose that the bidder is initially bidding for A , but that A is becoming increasingly expensive relative to B . At some point, if A becomes sufficiently more expensive than B , the bidder may want to switch to bid for B , which will trigger a corresponding reduction in its eligibility. From that point, the bidder's eligibility is insufficient to bid for A under the standard activity rule. However, suppose that subsequently the price for B increases more than the price of A , so that A is not so expensive relative to B . It is reasonable that in this case the bidder might again prefer A . Whilst this would not be possible under the standard activity rule, the revealed preference rule will allow for this, by allowing the bidder to bid for packages above its eligibility if such packages have become relatively cheaper than the packages for which it bid when reducing its eligibility in earlier rounds – so, in the example above, the bidder can bid again for A if the additional price of A relative to B is smaller than in the first round in which the bidder switched from A to B .

We do not recommend using an activity rule based on revealed preference unless a combinatorial auction is used, because this could introduce both the possibility and incentives for bidders to bid strategically on unwanted lots rather than the package of lots that it wishes to acquire at round prices where this could create opportunities for extending

demand. Bidders may be able to create a situation where they can bid above their eligibility by driving the price of lots for which they bid when reducing eligibility, so that bidding on unwanted lots can create more flexibility, as we illustrate in the following example.

Box 5: Strategic switching under a revealed preference activity rule

Suppose that a bidder is bidding for four lots of 700 MHz and, as it approaches its current budget, is considering reducing its demand to two lots. However, suppose for the sake of argument that the bidder is negotiating an increase in its budget and wants to keep some flexibility to bid back on four lots.

If the bidder drops straight back to two lots in the 700 MHz band, it will not be able to bid back on four lots. However, suppose the bidder could first reduce demand when switching to a different category, say 1400 MHz, and *then* switch to two lots in category 700. This would allow for the possibility that, if the price of two lots in category 700 MHz becomes *relatively* cheaper than the price of two lots in 1400 MHz than in the round in which it reduced demand, then the bidder might be able to increase its demand back to four lots in category 700 MHz.

More worryingly, the bidder might be able to provoke this situation if it wishes to increase its demand back to four lots, by bidding on 1400 MHz spectrum to drive its price up to the required level. If the strategy works, then winners of 1400 MHz spectrum might be overpaying for their lots; if the strategy does not work, then the auction could end with the bidder winning 1400 MHz spectrum, which is not what the bidder wants. Both possible outcomes are undesirable.

Withdrawal of standing high bids

Standing high bids create further switching impediments. When seeking to obtain several lots in the same band and holding standing high bids on some lots, a bidder is unable to switch all its demand in one go. For instance, suppose that a bidder wishes to obtain two lots of 2x5 MHz in either the 700 MHz band or the 2.1 GHz band. The bidder might initially bid for 700 MHz spectrum and become standing high bidder on only one lot. In the following round, it is possible that only the price of 700 MHz lots increases, so that at the new prices the bidder might prefer to switch to 2.1 GHz spectrum, which has become relatively cheaper. However, if the bidder is required to maintain its standing high bids it could at most switch one of its bids to 2.1 GHz. As the auction could end with the bidder winning the unwanted combination of one lot in each band, this creates risks that discourage switching.

One approach to mitigating such impediments is to allow bidders to withdraw their standing high bids. However, this makes bids no longer committing, giving bidders more opportunity to bid strategically. For this reason, withdrawals of standing high bids are often subject to constraints (e.g. there is only a limited number of withdrawals, or there are penalties for withdrawing standing high bids). However, such restrictions only impose costs on switching, reintroducing the inefficiencies associated with switching impediments without eliminating strategic abuse of withdrawals.

It is also possible to restrict the withdrawal of standing high bids to specific situations (for example, that the bidder is standing high bidder on only some of the lots it bid for, and that it withdraws its standing high bids in order to switch its whole demand to a different band). However, it is difficult to establish conditions that will effectively allow switching for legitimate reasons whilst preventing the strategic abuse of withdrawals.

In the upcoming auction we propose that standing high bidders are selected by satisfying bidders' demand in turn, which means that at most one bidder in lot category could end up as a standing high bidder on fewer lots than it bid for. Moreover, we understand that the expectation is for bidders to seek capacity in several bands, and to have decreasing marginal valuations if they can achieve a minimum bandwidth. All these factors limit the scope for inefficiencies arising from bidders being held by standing high bids when they would ideally switch. **Therefore, we do not consider that withdrawal of standing high bids is essential for this auction.**

Waivers

Waivers allow a bidder to skip a round without losing eligibility. If a bidder uses a waiver, then its eligibility for the following round is maintained. Waivers are commonly used in SMRAs, with bidders typically being given a limited (small) number of waivers.

Waivers address some of the inefficiencies arising from the potential switching impediments caused by standing high bidders. Using a waiver allows bidders to wait and see if they are outbid before switching their demand.

On the other hand, waivers may also be seen as a tool that facilitates collusion amongst bidders, in the sense that a bidder may reduce its own demand and place a waive, thereby testing the possibility of reducing demand whilst retaining the ability to bid back on more spectrum if the demand reduction is not matched by other bidders. However, we note that these concerns may be somewhat overstated, as a bidder would not know whether any observed reduction in demand from another bidder (assuming that this information would be provided at all) is genuine or equally backed by a waiver and thus reversible unless information about waivers placed is also disclosed (which we would not propose to do).

When aggregate demand information is not provided or is limited, bidders may also try to use waivers to gain more information about aggregate demand, as the waiver will allow the bidder to refrain from bidding in a given round and to observe how prices or other indicators change on the basis of competitors' demand only. However, using waivers for this purpose can be discouraged by giving bidders only a few waivers.

Waivers also provide a safeguard against potential problems with bid submission, where bidders might otherwise lose eligibility or be eliminated from the auction. As an alternative to

waivers in this regard, one could consider round extension rights, which give the bidder the option to extend a round for a limited time on a limited number of occasions if no bid is placed within the scheduled round time. However, such round extensions will only afford limited protection – with waivers, the auctioneer could pause the auction in case of substantial technical problems outside of the control of a bidder until these are resolved, whilst round extensions only give the bidder a little more time to solve any issues it might encounter (unless there is a backup bidding option available). Round extension will also not help with switching impediments resulting from standing high bids.

Thus, round extensions may be preferable in clock auctions, where the use of a waiver would interfere with auction mechanics (as it is not possible to determine aggregate demand if one or more bidders use waivers), whilst waivers may be the more appropriate instrument in SMRAs as they address both:

- switching impediments caused by standing high bids; and
- the need for a safeguard against possible problems with bid submission.

Therefore, for the upcoming auction we recommend allowing bidders to use a small number of waivers.

We see no benefit in combining waivers and extension rights, and we see no justification for restricting the availability of waivers to specific bidders (e.g. bidders who have some standing high bids).

Summary of recommended activity rules

In summary, we recommend that:

- lots are given eligibility points in proportion to their total bandwidth (potentially using downlink bandwidth only, which would affect the relation between SDL spectrum and the paired frequencies);
- a strict activity rule be used, where bidders' eligibility in a round is set to their activity in the preceding round (unless a waiver is used);
- withdrawal of standing high bids is not allowed; and
- bidders are given a limited number of waivers.

4.3 Information policy

Information disclosed during the bidding process

The benefits from using an open auction format are closely linked to the information disclosed to bidders. However, whilst on the one hand providing more information will contribute to bidders being able to refine their expectations about the outcome and potentially update their estimates of the value of lots, information about competitors' bids also facilitate undesirable bid strategies and tacit collusion.

We do not consider that it is necessary or desirable to disclose information about bidders' individual bids. However, it is reasonable to provide aggregate demand information, unless participation in the auction is severely limited and there are concerns of tacit collusion or other gaming strategies. Knowing total demand by band is helpful in terms of supporting outcome discovery and will improve bidding and auction efficiency. As noted above, concerns about the risk to realistic revenues from bidders reducing demand to keep prices low is best addressed by setting appropriate reserve prices which limit the potential gains from demand reduction relative to the benefits from obtaining additional spectrum where the incremental value exceeds price.

Knowing total demand will also minimise any informational advantage that bidders with partial standing high bids could obtain in a specific round. Such bidders can place a lower bound on total demand based on the number of lots in their bid on which they have not been designated as standing high bidders, which is information that would not be available to bidders who have the entirety of their bid designated as standing high bids. Although our proposed random ranking of bidders does not bestow any systematic informational advantage on specific bidders, reducing the degree of any information asymmetry, however small and short-lived, would nonetheless seem to be desirable.

If there are substantial concerns about the risk of collusion in the face of limited participation, less information might need to be given to bidders, ultimately limited to whether there is any excess demand (which would in any case be obvious from increasing prices). It is worth noting, however, that withholding information about the level of aggregate demand will not remove the incentives for bidders *unilaterally* to limit their demand to what they would consider acceptable at very low prices rather than trying to compete for additional bandwidth.

At the same time withholding information can sometimes trigger bid strategies aimed at probing competitors' demand, with bidders switching across categories not in response to price differences but simply to observe if prices continue to increase when they reduce demand in a given category. Such strategies distort price signals and could lead to an inefficient outcome if bidders fail to anticipate the end of the auction and do not switch back to the lots they do wish to acquire.

Considering this, ***we recommend that aggregate demand for each band is disclosed, but not the individual bids from each bidder.***

Implications of withholding aggregate demand information

If information about aggregate demand were to be withheld, however, then we would suggest that standing high bids be established by randomly ranking bids rather than ranking bidders. This is because in this case the additional information revealed to bidders who become standing high bidders on only

some of the lots for which they bid would effectively go against the objective of limiting information available to bidders.³⁴ If standing high bids are established on the basis of a random ranking of bids rather than bidders, and there may therefore be multiple bidders who hold standing high bids on only part of their demand in any band, waivers become more valuable in terms of addressing switching impediments, which in turn reduces the risk that waivers are used for strategic purposes.

Information disclosed after the auction

Disclosing information about all bids received after the process increases transparency of the process, allowing third parties to verify the correct running of the auction and understanding competitors' strategies. Bid data may contain sensitive information, especially if bidders bid close to valuations as would be the case in a sealed bid auction with a second price rule. However, some of this information (especially information about the highest losing bids) is already contained in the auction prices. In any case in a first-price iterative process it is not necessary for winners to bid up to their valuations.

Therefore, ***we consider that there are no strong arguments to withhold bid data SMRA stage after the auction.***

4.4 Assignment of specific frequencies

Several options exist for the assignment of specific frequencies, ranging from negotiations amongst winners with a regulatory backstop to ranking winners according to some criterion and allowing them to pick from a range of assignment options.

For this award, we would recommend to re-use the second price sealed bid combinatorial approach that has been used in many spectrum auctions, including the recent auctions in the Netherlands. This process has worked well and gives bidders the opportunity to express their preferences over all possible frequency assignments on an equal basis in the confidence that they will only be required to pay for a particular assignment if their preferences clash with those of another bidder and they displace the other bidder. By contrast, approaches in which bidders are permitted to pick their preferred assignment sequentially, even if the order in which bidders are invited to pick is determined based on bids, does not support a proper comparison of valuations and is strategically more complex. We can see no reason to deviate from the approach used in previous auctions.

Overview of the process

Using this approach, the assignment of specific frequencies is typically determined separately for each band even though bids

³⁴ Provided that there are no legal disclosure requirements that would conflict and that disclosure of the identity of applicants is not necessary to allow bidders to assess potential connections between them, for reasons of consistency it would also be desirable to withhold information about the number and identity of bidders prior to the commencement of the auction

for all bands are collected simultaneously. However, we understand that there are potential value interdependencies between the specific assignments that a bidder obtains in the 700 MHz and the 1400 MHz band resulting from passive intermodulation issues. Though dependent on equipment and filtering used, this may cause some MNOs to prefer specific combination of lots in the 700 MHz and 1400 MHz bands. In order to address this issue, we could either run the assignment processes sequentially, informing bidders about their assignments in the 700 MHz band before they must make bids for specific frequencies in the 1400 MHz band, or present joint assignment options for the two bands. We consider that the latter provides more scope for bidders to express their preferences, and therefore propose to use this approach. This means we would collect assignment bids for assignments in the 2100 MHz band, and for combinations of assignments in the 700/1400 MHz bands.

The starting point in each case is to shortlist, for each band, only those band plans in which each of the winners obtains a contiguous assignment that corresponds to the total bandwidth across the frequency-generic lots it has been assigned in that category and where any unassigned spectrum is also contiguous. This produces a number of 'candidate plans' for each band. The list of assignments that a bidder may receive in any of the candidate plans constitutes the relevant assignment options for the bidder. For the 700 MHz and 1400 MHz bands, the assignment options for each bidder would be all possible combinations of assignments in either band.

If one or more bidders have more than one option, a sealed-bid, second-price bidding process will be run. This allows bidders to express their preference for the different options available to them. The candidate plan that achieves the highest value will then be selected as the winning band plan, and bidders obtain their respective assignments.

In the first step, the 'candidate plans' for each of the bands will be identified. These are the band plans that are consistent with:

- assigning to each bidder the number of contiguous blocks in the band that corresponds to the number of frequency-generic lots the bidder has been assigned;
- any unassigned blocks are also contiguous.

These possible assignment plans will then be mapped into assignment options for each bidder, i.e. each bidder will be presented with all specific assignment that the bidder could receive in at least one of the candidate plans. The assignment options for the 700/1400 MHz bands are all possible combinations of the assignments that a bidder could obtain in either band.

Bids

Each bidder with different assignment options in a band/combination of bands can (but is not required to) make (mutually exclusive) bids for its different options in that

band/combination of bands. Each bidder is guaranteed to win one of its assignment options, regardless of whether it makes any bids.

A bid expresses the maximum price that the bidder would be willing to pay for being assigned that specific frequency range rather than any of the alternatives. Bids must be either zero or positive. By default, and if a bidder does not make a bid for an option, the bid for a frequency option is zero.

Winner and price determination

Bids are submitted simultaneously for all bands/combination of bands but will be evaluated for each band/combination of bands separately.

The 'value' of a candidate plan is calculated as the sum of bids for the assignment options in the plan.

The 'winning plan' for each band/combination of bands will be the candidate plan that achieves the highest value across all the candidate plans for that band. If there are more candidate band plans generating the same highest value, one will be picked at random. The 'winning bids' are the bids made by bidders' for the option they get in the winning plan in each band.

Each bidder will be required to pay a price for being assigned its winning options, which is calculated separately for each band/combination of bands, using a second pricing approach, i.e. bidders pay the lowest amount they could have bid for the assignment options they obtain without changing the outcome. This means that where the preferences of bidders do not conflict, each bidder will obtain its preferred assignment without having to make any payment.

The following example illustrates this process.

Box 6: Assignment bids and prices

Suppose that in the 700 MHz band three bidders (A, B and C) have each won two lots, The candidate band plans are created by looking through all possible bidder orderings (ABC, ACB, BAC, BCA, CAB, CBA) and identifying the blocks that bidders would receive in each of these. It is easy to see that each bidder could obtain either the lowest two blocks, the middle two blocks or the highest two blocks.

Now suppose that we receive the following bids:

- A bids €1,000 on the lowest two blocks, and blocks, and €500 on the upper two blocks.
- B bids €2,000 on the lowest two blocks, and €1,800 on the middle two blocks
- C bids €1,000 on the upper two blocks.

The values of the candidate band plans are thus as follows:

- ABC: €3,800
- ACB: €1,000
- BAC: €3,000
- BCA: €2,500
- CAB: €0
- CBA: €2,300

Band plan ABC generates the highest value and will therefore become the winning band plan.

In terms of pricing, we establish the best outcome that could be obtained if each of the bidders did not have any preference. If A had placed bids of zero on all options, the corresponding values would be:

- ABC: €2,800
- ACB: €0
- BAC: €3,000
- BCA: €2,000
- CAB: €0
- CBA: €1,800

The best option would now be band plan BAC, and A's opportunity cost is given by the difference between the value of this band plan (€3,000) and the bids of other bidders in the winning band plan (€2,800), i.e. €200.

- ABC: €2,000
- ACB: €1,000
- BAC: €1,000
- BCA: €500
- CAB: €0
- CBA: €500

ABC would continue to be the best option. B's opportunity cost is given by the difference between the value of this band plan (€2,000) and the bids of other bidders in the winning band plan (€2,000), i.e. B's opportunity cost is zero.

Repeating this exercise for C shows that C's opportunity cost is equally zero.

If neither A nor B had expressed any preference, the optimal band plan would have been either ABC or BAC with a value of \$1,000. The joint opportunity cost of A and B are therefore zero.

Without any preference from A and C, B would have obtained its preferred position (BAC or BCA) yielding 2000, so the joint opportunity costs of A and C are 200.

The joint opportunity costs of B and C are again zero.

This means that assignment prices are as follows:

A pays €200, which corresponds to the value lost from pushing B from its most preferred assignment to the second preference. B and C each pay nothing, as their preferences do not conflict with each other and B yields to A.

Annex A Overview of candidate auction formats

A.1 Simultaneous Multi-Round Ascending Auction (SMRA) formats

A.1.1 The standard SMRA

The key feature of the SMRA is that it evaluates bids on a lot-by-lot basis and builds upon the concept of standing high bidders.

This makes the bidding process intuitive, as bidders only need to improve their offers in response to being outbid. Selecting standing high bids also means that lots that received any bids will be sold.

Box 7: Overview of the SMRA bidding process

The bidding process works as follows:

- bids apply to specific lots;
- the auctioneer announces the price for each lot at which bids can be made in a round (which is the reserve price for lots that have not received bids and the highest bid received on the lot plus an increment for lots that have received bids) and bidders specify the lots for which they wish to bid at the prices announced by the auctioneer;
- at the end of the round, the auctioneer selects the highest bid on each lot (with random tie-break amongst bids of equal price), which becomes the 'standing high bid';
- at the end of the round, bidders are informed of the highest bid received on each lot, and of the lots on which they hold the standing high bid; and
- if any bids were received in the round, another round is run; otherwise the auction ends and the standing high bids become winning bids, with winners required to pay the amount of their bids.

If another round is run, bidders can place bids at the higher price, subject to an activity rule that requires that a bidder may not increase its demand relative to the preceding round. In the simple case where the lots offered are perfect substitutes this rule can simply require that a bidder cannot bid (or hold standing high bids) for more lots than in the preceding round. However, if the lots are imperfect substitutes then each lot is given a specific number of eligibility points, and demand is calculated as the sum of eligibility points associated with the lots for which the bid has bid or held a standing high bid.

If all the lots offered in the auction are perfect substitutes (e.g. lots in different bands), and there is no complementarity between groups, then an SMRA works very well, allowing bidders to revise their bids in response to being outbid.

However, standing high bids can create some problems when lots are not perfect substitutes:

- limiting the ability of bidders to switch across groups of lots; and
- exposing bidders to aggregation risks³⁵ in the case of synergistic valuations – as we show with an example below.

Switching impediments can be partially addressed by allowing for withdrawal of standing high bids. The rationale for withdrawals is to facilitate switching between different groups of lots.³⁶ However, the use of withdrawals complicates the auction, both in terms of mechanics and because it might allow bidders strategically to use withdrawals to distort the auction process in their favour. To mitigate the risk of such behaviour, withdrawals are usually subject to limitations on the number of occasions on which standing high bids can be withdrawn, or penalties (though this limits the usefulness of withdrawals in the first place), or to conditions that link withdrawals to the placement of new bids. However, even with provisions for withdrawals some switching impediments may remain.

*Aggregation risks
in an SMRA*

Aggregation risks affect bidders who have synergistic valuations. In very simple terms, suppose that at some given prices a bidder wishes to acquire either 2x10 MHz or nothing. In an SMRA the bidder must bid for each lot separately and may then become standing high bidder on only one of these lots. If the price for a second lot increases above the bidder's valuation, then the bidder may have to drop out, winning only one lot. We illustrate this with the following example.

³⁵ The risk of winning only some of the lots for which the bidder was bidding, at a price that exceeds the valuation for the lots won. An example is provided below.

³⁶ For instance, suppose that there are four lots, A, B, C and D, and a bidder was interested in either lots A and B, or lots C and D. Suppose that the bidder initially bids on A and B and becomes standing high bidder on A but not B. Suppose that in the following round, prices change and the bidder would prefer C and D. As the bidder holds the standing high bid on A it cannot simply switch to bidding for C and D. If withdrawal of standing high bids is allowed, then the bidder can withdraw its bid on A, in order to be free to bid on C and D.

Box 8: Aggregation risks in an SMRA

Consider the case of two bidders with the following valuations competing for three lots:

	Bidder A	Bidder B
One lot	11	4
Two lots	20	16

Marginal valuations are decreasing for Bidder A, i.e. the second lot is worth less than the first (9 for a second lot compared with 11 for the first lot). By contrast, Bidder B's marginal valuations are increasing (12 for a second lot compared with 4 for the first lot). If we assign two lots to Bidder A and one lot to Bidder B, we obtain a total value of 24; conversely, if we assign one lot to Bidder A and two lots to Bidder B, we obtain a total value of 27. Therefore, the efficient assignment is to give one lot to Bidder A and two to Bidder B.

In an SMRA Bidder B is exposed to aggregation risk:

- If it bids on two lots beyond a price of 4 per lot, it faces the prospect of ending up winning a single lot and having to pay a price that exceeds its valuation for the lot. For example, suppose that bids reach 8 per lot. At that point both bidders could still pursue two lots. However, Bidder A would continue to bid for two lots when prices are 9 per lot. If Bidder B only stops bidding for a second lot when prices reach 9 per lot, then it will end up with a single standing high bid of 8, which exceeds its valuation for a single lot. Alternatively, Bidder B might continue to bid for two lots in the hope of winning two lots for 18, which would also exceed its valuation but would imply a smaller loss than when winning a single lot for 8. The auction revenue in this case would be between 24 and 27, but Bidder B would be at a loss.
- Bidder B could stop bidding for a second lot as soon as prices are 4 per lot. This would ensure that the bidder is not exposed to the risk of overpaying for a single lot. However, the result would be unsatisfactory for Bidder B, as at this price it would much prefer to acquire two lots. The auction revenue in this case would be around 12.

In an SMRA, bidders facing aggregation risks are at an inherent disadvantage, which is likely to make them bid more cautiously and might even discourage them from taking part in the auction altogether.

Aggregation risks are a problem in the SMRA that cannot be easily fixed:

- Allowing bidders simply to withdraw their standing high bids would enable them to make bids that they do not intend to honour, possibly to drive up the prices paid by competitors, or to deny spectrum to competitors by making lots too expensive. The problems arising from bids not being committing can be severe.
- In some auctions, bidders can specify the minimum spectrum endowment that they would be willing to accept, so that if they end up standing high bidders on less than this required amount the bidder's bids will be cancelled. However, this rule requires that the bidder must commit to not being able to acquire a single lot before being able to assess the level of competition in the auction.

In addition, addressing aggregation risks in the SMRA may lead to some lots going *inefficiently* unsold. It is possible that lots may remain unassigned following the cancellation or withdrawal of standing high bids, whilst some bidders might have wanted to acquire such lots (even if at a lower price). However, such bidders may now be unable or unwilling to bid for the lots, as the current price of lots might already exceed their valuation.

A.1.2 The SMRA with augmented switching (SMRA-AS)

This variant of the SMRA is aimed at mitigating switching impediments that arise from the notion of standing high bids. The format was first implemented in Norway and has been subsequently used in Sweden, Finland (with some problems) and Poland (with some different problems).

As with the standard SMRA, bidding takes place over multiple rounds, with all lots sold simultaneously and the auction closing on all lots simultaneously when there is no new bidder activity. However, unlike in the standard SMRA auction, standing high bids can be withdrawn, if they are switched to bids on other lots.

Thus, standing high bids can be withdrawn in order to switch to alternative lots, but not in order to reduce demand. This addresses the problem in the standard SMRA that standing high bids create impediments to switching between different groups of lots but does not mitigate aggregation risks.

The lots from which standing high bids have been withdrawn could in principle be designated as not having any standing high bidder. However, this has caused concerns that some lots may remain unsold at the end of the auction even though they had received bids in some previous round. For this reason, there are typically provisions to ensure that in case of withdrawal of a standing high bid the previously second-ranked bid (which could be a previous standing high bid or another bid submitted in the same round as the standing high bid but ranked below on the basis of price or as a result of a tie break) will become the standing high bid.

This means that bids on lots that are not the standing high bid in any given round can become the standing high bid in the further course of the auction, even if the bidder had stopped reduced eligibility (reactivation of bids would restore bidder eligibility when required). Any bid made during the auction can effectively be reactivated (unless it has been withdrawn), and therefore bidders are committed to *all* their bids made throughout the auction, regardless of whether they are standing high bids. Even a bidder who has reduced demand and potentially dropped out of the auction completely may find itself winning one of the bids it made in earlier rounds, which increases exposure to aggregation risks and can make

managing budgets challenging.³⁷ In order to manage this, and to ensure that bidders cannot be reactivated on more lots than the bidders would have been initially eligible to win, bidders who switch without being standing high bidder on the respective lots (or who switch more eligibility than that they have on standing high bids) also need to specify which of their bids (not standing high bids) they wish to withdraw in order to bid on other lots.

It is worth pointing out that re-activation of previous bids is not essential for using the more permissive withdrawal rules that allow withdrawal of standing high bids for the purpose of switching. The rules proposed by the Swedish PTS for the 700 MHz award did not include provisions for re-activation of earlier bids, but instead limited withdrawals to situations where they were necessary in order to enable switching between groups of lots.

Where augmented switching rules are used and prices of lots on which standing high bids can be withdrawn, it is crucially important to ensure that bidders who have withdrawn a standing high bid from a particular lot will not be able to bid back on that lot in future rounds at a lower price. This ensures that even if the amount of the standing high bid falls as a result of successive re-activations of earlier bids, and other bidders may then place bids on that lot at a lower amount, the bidder who has withdrawn the standing high bid does not benefit from such a reduction in prices.

This crucial provision had been overlooked in the Finnish implementations of this format for the 2.6 GHz auction. Where in this auction the oversight only resulted in prices dropping down to reserve (or slightly above) after 27 rounds of bidding, the problem became evident in the 800 MHz, which ran for eight months with prices going up and down without getting any closer to resolving excess demand. The Finnish regulator suspended the process claiming that there was a need for software maintenance, and in the process specified that prices could not drop back to the reserve price but made no other changes. Nevertheless, bidding came to an end shortly after the auction resumed.

For the Finnish 700 MHz, the rules were amended to stipulate that bidders could not bid back on lots from which they had withdrawn a standing high bid at a lower price.

³⁷ For this reason, the rules may include a provision for a minimum level of eligibility to which the eligibility of a bidder is increased if bids are reactivated in order to ensure that a bidder who had lost eligibility before having one or more of their bids reactivated has enough eligibility to bid for sufficiently large combinations of lots to win usable spectrum.

A.1.3 The SMRA with hierarchical package bidding (SMRA-HPB)

This variant of the SMRA was intended to address concerns about aggregation risks facing bidders whose business plans rely on being able to acquire combinations of lots. It introduces limited package bidding into a simple SMRA design by defining non-overlapping groups of lots on which bids can be placed in addition to standard bids on individual lots. It is then relatively simple to compare the aggregate bid amounts on the individual lots with the bid amount on the package in order to determine whether the package bid or the bids on individual lots should become standing high bids. It is even possible to define a hierarchy of packages, where at each level of the hierarchy the packages defined do not overlap, and collectively exhaust the next larger package. An example of such a hierarchical structure is shown below.

Figure 2: Example of a hierarchical package structure

A	B	C	D	E	F	G	H
AB		CD		EF		GH	
ABCD				EFGH			
ABCDEFGH							

This approach was used for the US 700 MHz auction, where one of the blocks was available on a regional basis or in the form of three packages – one covering the 50 main US states, one covering the US Pacific territories and the last one including Puerto Rico, the U.S. Virgin Islands and the Gulf of Mexico).

Minimum bid amounts were set for individual lots, and where the standing high bid is a package bid above the sum of lot prices, there would be implied lot prices scaled up to meet the package bid amount for the purpose of setting the next bid levels.

Even in this simple two-level setting, there are issues that complicate the standard SMRA design beyond simply defining packages. For example, in order to determine whether the package bid or the individual lot bids should be accommodated, bids submitted on individual lots in prior rounds that were not standing high bids but would mesh with the bids submitted in the most recent round were considered, which could then result in bidders winning bids on lots that they might not have been interested in acquiring any more (and potentially being active on lots in excess of their eligibility). This then required provisions

that allowed bidders to drop such bids without detrimental effects (e.g. no penalties unlike in the case of standard withdrawals).

Also, withdrawal of bids on lots that were also part of packages was not permitted, as such withdrawals would have repercussions for the determination of winning bids.

Experiments conducted by the designers of this format suggest that hierarchical package bidding performs better than flexible package bidding and the standard SMRA in terms of efficiency, unsold lots and revenues.³⁸ It is not clear, however, to what extent these findings can be generalised and the authors acknowledge that the performance depends on being able to define packages that match bidders' preferences sufficiently closely. However, in many practical applications this limits the use of the HPB approach as the relevant packages are either unknown to the auction design, or else do not fall into a hierarchical, nested structure. It is also worth pointing out again that in the presence of complementarities unsold lots do not *per se* give rise to concerns, and that auction formats that are aimed at assigning all spectrum regardless of whether some lots might efficiently remain unsold expose bidders to risks that can distort bidding behaviour and result in inefficiencies that go beyond assigning lots that should remain unassigned.

A.2 Clock auctions

A.2.1 Simple clock auction (SCA)

Clock auctions are well-suited for offering groups of identical items (such as frequency-generic blocks of spectrum).

³⁸ Goeree, J. K. and Holt, C. A. (2007). Hierarchical Package Bidding: A Paper & Pencil Combinatorial Auction, mimeo, available at <http://www.people.virginia.edu/~cah2k/simpleCA.pdf>

Box 9: Overview of the clock auction bidding process

The simple clock auction works as follows:

- identical lots are grouped together into lot categories;
- the auctioneer announces the price for each lot category in a round, and bidders specify the number of lots in each category they wish to acquire at the prices announced by the auctioneer;
- if there is excess demand for any of the lot categories (i.e. if the total number of lots that bidders indicated they wish to acquire at the round price exceeds the number of lots available), then a further round needs to be run, with a higher round price for lot categories that had excess demand; otherwise the auction ends and each bidder is given the lots it specified it wishes to acquire at the round price.

Clock auctions usually use the same activity as the SMRA: lots are given eligibility points, and demand (calculated as the sum of eligibility points associated with the lots for which the bid has bid) cannot increase relative to the preceding round.

Clock auctions allow for easier switching and do not expose bidders to aggregation risks as there are no standing high bids on a subset of the lots on which a bidder placed bids. In that sense, bids submitted in the clock auction are package bids – a bidder either wins her clock bid in its entirety, or not at all, but never faces the risk of winning a subset of the lots in the clock bid.

Clock auctions can also resolve excess demand much more quickly when there are many substitutable lots, as these are grouped into lot categories to which price increments apply uniformly.

However, the clock auction typically uses the same activity rules as the SMRA, where bidders cannot increase their demand relative to the preceding round, and thus switching impediments from eligibility-points based activity rules will remain. As in the SMRA, these impediments, can be mitigated by a relaxed activity requirement, but this is only a partial solution.

A downside of clock auctions is that the flexibility afforded to bidders in terms of being able to switch all their demand in one go can give rise to coordination problems. This will happen for instance if several bidders who are indifferent between two categories switch at the same time. This problem is reinforced by the price signals provided in the clock auction. We show this in the following example.

Box 10: Coordination problems in a clock auction

Suppose that we have two lots in each of two categories, and two bidders who want two lots each in one category, but they do not particularly care about which category (or who have a very mild preference for one category). If in the first round both bidders bid for the same category, then there will be excess demand for that category and not the other, so that the price will only increase for one category. In response, both bidders might switch to the other category. This again leads to excess demand in one category but not the other, and to the price increasing in only one category. This alternate increase of prices can continue until one of the bidders stops switching or reduces its demand, possibly resulting in some lots unsold in one category. However, it would have been perfectly possible to accommodate both bidders in the first round.

In a simple clock auction, there is also a risk that demand might drop too abruptly from one round to another (e.g. if several bidders reduce demand in the same round, or if bidders reduce demand by several units in one step). Thus, we might go from a situation in which there is excess demand to a situation in which the auction ends with unsold lots. Such large drops in demand may be the result of price increments being too large or arise from synergistic valuations.

The first cause for large drops in demand can easily be addressed by allowing (or requiring) bidders to make *exit bids* when they reduce demand. These exit bids would be the best offer that a bidder makes for lots on which it ceases to bid. Exit bids specify a price (required to be between the round price in the preceding round and the current round price)³⁹ at which the bidder would be prepared to buy the lots she no longer demands at the current round price. For example, if a bidder reduces demand from five to two lots, she would specify the price at which she would still demand four lots, and the price at which she would be happy to buy three lots. Then, if the auction were to end with excess supply in any lot category, the auctioneer could look into accepting one or more of the exit bids in that category, in which case the price per lot for the category would be dropped to that of the lowest exit bid accepted.

By contrast, there is no easy solution for addressing the problem of unsold lots if there is a large drop in demand because of synergistic valuations. Synergistic valuations may result in unsold lots because bidders do not wish to reduce demand progressively and might therefore not be willing to make exit bids for the different intermediate quantities. We illustrate this with an example below.

To mitigate the risk of unsold lots we can **require** exit bids, with the additional provision that these exit bids may be honoured only partly: for example, if a bidder reduces demand from five lots, say, to two lots, and makes an exit bid for four lots, this

³⁹ For example, suppose that at round prices of 10 per lot a bidder is bidding for three lots. In the following round, when the price increases to 11 per lot the bidder decides to bid for two lots. The bidder could then make an exit bid for a third lot at a price between 10 and 11.

would be taken as a willingness to acquire **up to** four lots at the price specified, even though the bidder may not wish to win three lots at any price above the previous round price. Therefore, this reintroduces aggregation risks by effectively forcing bidders to bid for subsets of lots at the price per lot they offered for a greater number of lots.

Box 11: Unsold lots in a clock auction due to synergistic valuations

Consider the same set-up as above, i.e. two bidders with the following valuations competing for three lots, but now with a clock auction instead of an SMRA:

	Bidder A	Bidder B
One lot	11	4
Two lots	20	16

In a simple clock auction (exit bids may be allowed, but not required) bidders do not face aggregation risks, as they can simply withdraw their full demand from one round to another. Therefore, Bidder B could bid for two lots until the price reaches 8, and then exit cleanly. However, this would leave one lot unsold, and Bidder B without any spectrum. This is obviously far from ideal (the auction revenue in this case would only be 18).

Alternatively, one could require exit bids, so that the Bidder B would not be able to drop down demand from two lots to nothing. However, this would reintroduce aggregation risks: if the bidder continues to bid for two lots until the price per lot is 8, then it will be required to also bid for a single lot at a price of 8 or higher. Therefore, the bidder may stop bidding at a lower price. Indeed, if the bidder wants to completely avoid the risk of overpaying it will stop bidding when the price is 4. The auction revenue in this case would be 12.

A.2.2 Clock-Plus auction (CA+)

However, exit bids alone do not fully address the risk of unsold lots if there are multiple lot categories. For example, bidders may switch without any reduction in activity and would therefore not be reducing demand yet leave the category from which the switch with excess supply. Even if bidders were subsequently required to make exit bids as and when they reduce demand, these bids would not ensure that all the lots in the first category will be sold.

To address this problem, some variants of the clock auction (so-called 'Clock-Plus' format as used for recent auctions in Singapore or for the forward auction of the US 600 MHz incentive auction) impose switching restrictions that limit the extent to which switches will be accommodated if they were to leave a particular lot category with excess supply. This means that bidders may end up with subsets of lots in which they are interested (e.g. if they cannot withdraw demand cleanly, but are required to take up some lots that would otherwise remain unsold) and with split assignments (e.g. if they wanted to switch a number of blocks from one band to another, but that switch

was only partially accepted). The implementation of these restrictions also results in fairly complex rules in terms of measuring activity, and in the case of more than two lot categories also requires that bidders specify preferences for partially accepted switches, which would seem to make the process much more complicated than the SMRA-clock hybrid format.

More generally, the reason why efficiency cannot be guaranteed in the clock auction is that it requires using linear prices (all lots in a category are sold at the same price). However, in order to assign all the lots, we may need to assign those lots for which there is no demand at the final clock prices at a lower price (where the price *per lot* might depend on the number of lots acquired). In order to support this, bidders would need to be allowed to bid for different ‘packages’ (i.e. number of lots) at amounts that imply different prices per lot.

A.3 The SMRA-clock hybrid

Requiring exit bids and restricting switching or withdrawals makes the clock auction more SMRA-like without requiring the notion of standing high bids and standing high bidders. However, as these constraints expose bidders to the same risks and limitations that arise from the notion of standing high bidders in the SMRA, there is little gained from not explicitly declaring such standing high bidders whilst retaining the pricing mechanism of the clock auction in a hybrid format.

Typical hybrid rules

In such hybrid formats:

- The auctioneer specifies clock prices for each lot category, and bidders specify the number of lots for which they wish to make bids at that price in each category.
- At the end of each round, the auctioneer selects standing high bidders in each lot category – in order to minimise the number of bidders who may receive fewer lots than they bid for, this can be done by ordering bidders (using pre-specified criteria or at random) and satisfying their demand in turn until there are no more lots available, so that at most one of those bidders who is standing high bidder will be standing high bidder on only a subset of the lots requested.
- If all the lots in a category have a standing high bid at the clock price, then the clock price for the following round is increased.
- Standing high bidders can keep their standing high bid unchanged, or increase it to a higher clock price, and activity rules apply as in an SMRA (both new bids and unchanged standing high bids count towards activity).
- The auction closes after a round in which no new bids have been received or withdrawals have been made (to the extent permitted).

This approach has the advantage of progressing more quickly if there is a small amount of excess demand for many lots, and, depending on the specific rules, producing uniform prices for similar lots rather than the (roughly) similar prices that typically emerge in an SMRA. It was used initially for the 3G and BWA auctions in India 2010 (and has subsequently been used, with small modifications, for further auctions in India⁴⁰), and has recently been used in the UK PSSR auction combined with provisions that allow bidders to opt in for a 'minimum requirement' in the 3.4 GHz band of 20 MHz⁴¹ and for withdrawals of standing high bids⁴² to address concerns about aggregation risks.

A.4 Combinatorial Clock Auctions

A.4.1 Standard Combinatorial Clock Auction (CCA)

A CCA is structured as a simple clock auction followed by a sealed bid combinatorial round in which bidders are subject to constraints that arise from the bids they have made during the clock auction. The rationale for this is that:

⁴⁰ The closing rule for the Indian 3G and BWA auctions differed from that in the SMRA in that the auction would close as soon as there was no excess demand at clock prices. Previous standing high bids that had not been raised did not count towards demand at clock prices, and thus bidders who waited until they were 'outbid' before raising their own bids faced the risk of not being able to come back if they were displaced by bids placed at the new clock price and the auction ended. This rule was designed to provide incentives for bidders to increase their standing high bids, reflecting that there was an explicit revenue objective associated with this auction. The closing rule could produce higher revenues because even if demand was exactly equal to supply, bidders could continue to, and might have an incentive to, increase their bids.

Bidders were informed about the level of demand for each lot category at the clock price (but not whether the demand came from raised standing high bids or from new bids). Subsequent auctions in India adopted a modified closing rule, by which the auction only ends when there are no new bids or waivers (rather than when there is no excess demand at clock prices). This alternative closing rule is more similar to that in an SMRA and allows provisional winners to wait to be outbid before increasing their own bid.

⁴¹ Any bidder who opts in for this minimum requirement and ends the auction being standing high bidder on less than 20 MHz in this band will have these standing high bids cancelled.

⁴² Bidders are allowed to make a limited number of withdrawals if they have become standing high bidders on fewer lots than they had bid for in a category and subject to the requirement that the bidder withdrawing standing high bids from a category must withdraw all of its standing high bids in that category and make no new bids for that category in that round. Bidders who withdraw standing high bids are still liable to pay for any bids they have withdrawn if the corresponding lots do not receive further bids.

- the final sealed bid round eliminates aggregation and substitution risks by permitting bidders to express a full demand profile in a list of mutually exclusive package bids; and
- the initial clock auction phase provides information to bidders about demand, thus reducing the uncertainty they will face in the final sealed bid round relative to a simple sealed bid auction.

Box 12: Overview of the CCA bidding process

The process starts as a simple clock auction, as follows:

- identical lots are grouped together into lot categories;
- the auctioneer announces the price for each lot category in a round, and bidders specify the number of lots in each category they wish to acquire at the prices announced by the auctioneer;
- if there is excess demand for any of the lot categories (i.e. if the total number of lots that bidders indicated they wish to acquire at the round price exceeds the number of lots available), then a further round needs to be run, with a higher round price for lot categories that had excess demand; otherwise the clock auction phase ends.

The clock phase is followed by a single round (the 'supplementary bids round') in which bidders can make additional, mutually exclusive bids for alternative packages (subject to constraints arising from the activity rules outlined below).

The winning bids are then selected from all the bids received in the auction. The selection of winning bids can adopt different bidding or outcome constraints and will typically use an opportunity-cost based pricing rule (like a second price rule in a single-item auction but adapted for the case of multiple items).

The more basic activity rules for the clock auction phase are identical to those in a simple clock auction, where demand (calculated as the sum of eligibility points associated with the lots for which the bid has bid) cannot increase relative to the preceding round.

Each time that a bidder reduces demand, this will impose a revealed preference constraint on the packages that the bidder would cease to be able to bid for under the basic activity rules.⁴³ These constraints, called 'relative caps', are applied in the supplementary round, and set a limit on the bid amount that the bidder can make for the package affected by the constraint that is defined in relation to the bids that the bidder makes for other packages for which it has bid during the clock rounds.

The CCA can adopt a relaxed activity rule for the clock phase, which will allow bidders to increase their demand (in terms of eligibility points) relative to the preceding round if doing so is consistent with the relative caps. Effectively this allows bidders to make bids that they would have been able to do in the supplementary bids round, and improves the information disclosed during the clock phase.

Additional revealed preference constraints arise in the final round of the clock phase. If the basic activity rule is used for the clock phase, these

⁴³ The revealed preference constraint is set with reference to choices made by the bidder. Specifically, suppose that a bidder selects package X over package Y when round prices are P_X and P_Y . A revealed preference constraint in relation to this choice would constrain the bid for Y to be at most the bid for X plus $P_Y - P_X$. This means that the bidder may need to raise its bid for X in order to also raise its bid for Y above the prices that applied when it made the constraining choice.

caps will only affect packages with eligibility equal to or lower than the bidder's bid in the final clock round. If the relaxed activity rules are used, then this cap affects all packages, effectively requiring that the bidder will have to satisfy revealed preference with respect to the final clock round in all its bids.

The CCA has several desirable features:

- there are no aggregation risks in a CCA, as bids are submitted for indivisible packages of lots;
- switching and coordination impediments are removed by allowing bidders to make a list of mutually exclusive bids;
- an efficient outcome is possible even if there are synergistic valuations, as prices are not bound to linear prices.

However, the mechanics of the CCA are clearly more complex than those of the SMRA or the simple clock auction. This can create discomfort for bidders and increase the scope for mistakes, especially if bidders try to second-guess competitors in order to bid strategically to distort the outcome in their favour.

Many bidders have also expressed discomfort with respect to the sealed bid aspect of the auction, which exposes to uncertainty about the lots they will eventually win and the price they may need to pay. The level of uncertainty is greatly reduced when the relaxed activity rules are used. However, these rules also increase the mechanical complexity of the process. Furthermore, it is still possible in some cases that a bidder who has made bids at the final clock round prices may eventually fail to win any spectrum at all after the supplementary bids round – such outcome is efficient in that it involves a better assignment of the available lots, but bidders who would leave empty-handed might then find that they would consider changing their bids to try to win some spectrum.

Bidders have also objected to the opportunity cost-based pricing rule used in the CCA on the grounds that:

- it creates governance problems for bidders, as the optimal bid strategy may require making bids that are much higher than what the bidder expects to have to pay eventually, but these decisions are difficult to make and sign off;
- it poses challenges who are subject to a relatively tight budget constraint, who will need to focus their bids according to their expectation of what they expect to be able to win within their budget;
- it provides incentives for bidders to make bids they do not expect to win in order to increase the price that competitors will have to pay; and
- can lead to asymmetric prices, which can be difficult for bid teams to justify to the company board and/or shareholders.

Despite these concerns, the CCA remains an efficient auction format that is particularly suited for multi-band auctions. It can be expected to perform very well if bidders have budgets that reflect their actual valuations and that they have the means to prepare for and bid during the auction.

There have also been attempts to address some of these issues through changing activity rules, e.g. using stricter revealed preference constraints based on the General Axiom of Revealed Preference (GARP), looking at all choices made by bidders rather than simply at the decisions in rounds in which they reduced eligibility. Such stricter GARP rules will be used by the Canadian regulator for the forthcoming 600 MHz auction.⁴⁴

A.4.2 Enhanced Combinatorial Clock Auction (ECCA)

In order to address some of the concerns raised in connection with the CCA (in particular the relevance of the outcome of the clock rounds for the final assignment of lots, and the scope for strategic bidding), proposals for a so-called Enhanced CCA ('ECCA') have been put forward. The most fully articulated description of the specifics of the rule can be found in the consultation document issued by the Canadian Department for Innovation, Science and Economic Development (ISED).⁴⁵ The core modifications relative to the CCA are that a stricter activity rule is used for both the clock rounds and the supplementary round, and a modified pricing rule.

The activity rule of the ECCA permits bidders to submit bids for package that exceed their eligibility, but only if all of the bids submitted by the bidder since the last round in which it had sufficient eligibility to bid on this package are consistent with truthful bidding based on some implied set of valuations. The requirement of consistency of all bid decisions from the round in which the bidder would last have been able to bid on a specific package based on its eligibility with an underlying set of valuations is also extended to supplementary bids. Overall, the activity rule of the ECCA imposes tighter constraints on the additional bid amounts that can be placed on larger packages.

Given these tighter constraints, the ECCA rules then determine prices using the highest valuations that bidders could possess, given the bids that they have made. Put simply, rather than using the bids actually placed by other bidders, the ECCA pricing rule considers the maximum bids that such other bidders could have placed for larger packages that incorporate the lots in a bidder's package (and eventually in the winning package), given those other bidders' bidding history.

In the clock rounds bidders will be informed before each clock round about the amount by which their base price would be lower than their bid, based on the choices made by other

⁴⁴ ISED, Technical, Policy and Licensing Framework for Spectrum in the 600 MHz Band, <https://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf11374.html>.

⁴⁵ ISED, Consultation on a Technical, Policy and Licensing Framework for Spectrum in the 600 MHz Band, Annex C, <https://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf11316.html#sC>

bidders so far. Should the clock round end without any unsold lots, prices would simply be determined by applying the respective discounts to the final clock bids (i.e. by calculating the most that other bidders could possibly bid for the lots won by a particular bidder) without the need for running a supplementary round. By implication, a supplementary round would only be needed if there were unsold lots at the end of the clock stage.

These modifications are intended to make the clock stage more relevant and limit the role of the supplementary round to assigning lots that might otherwise remain unsold and to reduce the scope for strategic bidding, thus improving price discovery.

A.5 Combinatorial Multi-Round Ascending Auction (CMRA)

The CMRA builds on the CCA with relaxed activity rules for the clock auction phase. However, the CMRA eliminates some elements of the CCA in relation to which bidders have expressed concern:

- it does not have a final sealed bid round – instead, it allows bidders to make multiple bids in each clock round (subject to the constraints that would apply to the supplementary bids round of the CCA) and runs a combinatorial evaluation of bids at the end of each round;
- it does not expose bidders to the risk of not winning any spectrum unless they explicitly stop making bids at round prices; and
- it uses a pay-your-bid rule instead of the opportunity cost-based pricing rule used in the CCA.

Box 13: Overview of the CMRA bidding process

The process follows the multi-round structure of a clock auction, in that:

- identical lots are grouped together into lot categories;
- the auctioneer announces the price for each lot category in a round, and bidders specify the number of lots in each category they wish to acquire at the prices announced by the auctioneer – this constitutes the **headline bid** of the bidder in that round.

However, bidders can also make **additional bids** in each round, subject to the constraint that none of these bids can exceed the round price, and that relative caps that arise from previous headline bids are satisfied. These relative caps arise when a bidder reduces its eligibility by bidding on a headline bid with less eligibility than its preceding one, following the same approach as in a CCA.

Another difference is that the auction does not end when there is no excess demand at round prices in any category, but rather when the optimal outcome given the bids received so far (using a combinatorial evaluation of bids analogous to that used after the supplementary bids round in a CCA) involves accepting a bid from each bidder – these become the winning bids and bidders pay the amount of their bid.

The closing rule differs from that in a clock auction in that the auction might continue even if there is no excess demand at round prices. However, this will only happen if any of the bidders who is still bidding at round prices would be outbid with the bids made so far. At the same time, it is also possible that the auction might end when there is still excess demand at round prices, if it is possible to accept a bid from each bidder by considering their additional bids.

The CMRA adopts the relaxed activity rules developed for the CCA, which allow bidders to increase their demand (in terms of eligibility points) relative to the preceding round if doing so is consistent with the relative caps. This allows bidders to make bids that they would have been able to do in the supplementary bids round of a CCA.

The CMRA has several desirable features in common with the CCA:

- there are no aggregation risks in a CMRA, as bids are submitted for indivisible packages of lots;
- switching and coordination impediments are removed by allowing bidders to make a list of mutually exclusive bids each round, and by allowing bidders to increase their demand in response to price movements;
- an efficient outcome is possible even if there are synergistic valuations, as prices are not bound to linear prices.

In addition, the CMRA provides greater control to bidders with respect to the possible outcome, by allowing them to progressively increase the number of packages they bid for as they need to. The CMRA also provides certainty about the price to be paid and does not require (or allow) bidders to make bids above round prices, ensuring that bidding is progressive and predictable.

The mechanics of the CMRA are clearly more complex than those of the SMRA or the simple clock auction. As with the CCA, this can create discomfort for bidders and increase the scope for mistakes, especially if bidders try to second-guess

competitors in order to bid strategically to distort the outcome in their favour.

The CMRA is also subject to the problems associated with the pay-your-bid rule, in particular:

- bidders may try to shade their bids (i.e. bid below their true valuation) with a view to maximising their surplus (i.e. the difference between their valuation and the price paid);
- bidders may have an incentive to reduce demand early in order to win some lots at a lower price.

However, the incentives to reduce demand in headline bids is (partly) mitigated through allowing bidders to make additional bids below round prices.

The outcome of the CMRA can be expected to be aligned with that of a CCA, and thus should perform well for multi-band auctions. It can be expected to perform very well provided even if some bidders must bid to a tight budget constraint, if they have the means to prepare for and bid during the auction.

A.6 Assessment of candidate auction models against the award objectives

Based on the description of the various possible auction formats and specific models, we now provide a brief assessment against the objectives (producing an efficient outcome, generating realistic revenues, being simple and transparent and providing bidders with freedom of choice) for this award. Our assessment is based on the understanding of the nature of demand – specifically the fact that there are no strong complementarities across bands and that any residual aggregation risks within bands should be manageable for bidders).

Sealed bid formats We have focused on open multi-round formats, mainly because sealed bid processes carry a substantial risk of inefficient outcomes because bidders may fail to place bids on efficiency-relevant combinations of lots and must make their bid decisions based on expectations about the behaviour of their competitors, which may be wrong. For this reason, we consider that the format **does not contribute towards achieving the efficiency objective**.

The format should **perform well in terms of achieving a realistic revenue** as it is fairly resistant to strategic bidding and has limited incentives for strategic demand reduction.

Though the rules are very simple, taking part in such an auction is strategically challenging for bidders and therefore we score the format as **not contributing towards the objective of simplicity**. By the very nature of such sealed bid processes they also **are not transparent**.

Although bidders might in theory be able to submit a large number of bids for different portfolios and therefore the format might be seen to **perform well in terms of freedom of choice in theory**, in practice bidders may find it difficult to place these bids without any information about what combinations of lots they might be able to win at different prices.

CCA/ECCA

These formats **perform very well in terms of achieving an efficient outcome**. Support for package bidding without the constraints that arise from the designation of standing high bids means that there are no switching impediments and that any residual aggregation risks are eliminated.

The second price rule employed in these formats eliminates the incentives for strategic demand reduction and makes tacit collusion difficult, but revenues are limited by the extent to which bidders can express valuation for spectrum that is incremental to their winnings. Overall, this suggests that the format **performs well in terms of generating realistic revenues**.

As with any format that supports full package bidding, the rules of the auction are inevitably complex, which creates its own challenges. For this reason, the format **does not contribute towards the objective of simplicity**. The sealed-bid component that is present in the form of the supplementary bid stage and the potentially complex activity constraints and the calculation of the discount in the ECCA imply that the formats also **do not score on the objective of transparency**.

Like all open multi-round processes, the formats **perform well in terms of providing freedom of choice**, allowing bidders to adjust their demand in light of observed price developments (though there may be constraints in terms of the number of different packages on which bids can be placed).

CMRA

The CMRA performs similarly to the CCA/ECCA formats. The combinatorial nature of the format means it **performs very well in terms of efficiency**.

Being a pay-as-bid format, there could be concerns about strategic demand reduction, but provided that these are being addressed through reserve prices set at an appropriate level, the format should perform well in terms of achieving a realistic revenue.

The format requires a complex set of rules and **does therefore not score against the objective of simplicity**. However, without exposing bidders to uncertainty over outcomes (unless they are prepared to place a set of additional bids) the CMRA **should perform well in terms of transparency**.

Like the other combinatorial formats, it also **performs well in providing bidders with freedom of choice**.

Simple clock auction

The simple clock auction permits bidders to switch between spectrum portfolios without any impediment that would arise

from the designation of standing high bids and also eliminates any residual aggregation risks but could result in efficiently unsold lots⁴⁶ and might encourage strategic bidding, as bids are not committing. Overall, in the simple form, the format **performs well in terms of achieving an efficient outcome**.

As in the case of the CMRA, the fact that that winning bidders pay the amount of their bid can create incentives for demand reduction, but again these incentives can be counteracted through setting appropriate reserve prices. In this case, the format **performs well in achieving realistic revenues**.

The format is **simple** – both in terms of the rules and the challenges for bidders – and **transparent**, so **performs very well against these two objectives**.

Bidders are free to choose their preferred packages at round prices but are not generally able to express the fact that they might also be interested in taking other combinations of lots, so we consider that the format **performs well in terms of freedom of choice**.

SMRA

Without synergistic valuations, the **SMRA format performs well in terms of achieving an efficient outcome**. Whilst the designation of standing high bids can create some switching impediments bidders are generally able to respond to changing prices and pick their preferred portfolios. With all bids being committing the risk of unsold lots is low and strategic bidding becomes more difficult than in the simple clock auction, for example.

In terms of **achieving realistic revenues**, the same considerations as in the case of the simple clock auction apply and the **format performs well** if reserve prices are set at an appropriate level.

Like the simple clock auction, the SMRA is **simple** and **transparent**, so **performs very well** against these objectives.

Similarly, the format **performs well in terms of freedom of choice**, with bidders being able to respond to changes in prices, though possibly somewhat constrained by standing high bids.

SMRA-Clock-Hybrid

The SMRA-Clock-Hybrid shares the characteristics of the SMRA and thus has very similar scores. As discussed, the switching impediments that result from standing high bids may be somewhat reduced by ensuring that at most one bidder in each lot category has only part of its demand tied up in standing high bids, and therefore the format can be said to **perform very well against the objective of achieving an efficient outcome**. It is worth emphasising, however, that this advantage over the standard SMRA would be lost if the designation of standing high bids were based on a random ranking of bids

⁴⁶ Without underlying synergies, the risk of unsold lots could be reduced through modifications of the format (namely the option for placing exit bids).

rather than bidders. In this case, the only advantage would be the greater procedural efficiency in terms of avoiding the 'long tails' typically associated with SMRAs with little excess demand for a number of homogeneous lots.

Clock-Plus

As the Clock-Plus auction does not provide the benefit of limiting the number of bidders who only have part of their demand in a lot category designated as standing high bids, but could limit switching for multiple bidders and without bidders knowing that they might be limited when attempting to switch, we consider that the format **performs well in terms of achieving efficiency**, but not as well as the SMRA-Clock Hybrid.

In terms of **achieving a realistic revenue**, the same considerations as for the other pay-as-bid formats apply – provided that reserve prices are set at an appropriate level, the format **performs well**.

We consider that the differentiation between the bids submitted and processed bids adds substantial complexity to the rules and makes the format somewhat less transparent (especially with more than two lot categories, where there is either a need for specifying switching preferences or a lack of control by the bidder). For this reason, we consider that although the format still **performs well in terms of transparency**, it **does not score in terms of simplicity**.

As the other open multi-round formats, the format **performs well in terms of freedom of choice** (though bids may not be fully accepted).

The table below summarises this assessment.

Objective	Sealed bid	CCA/ECCA	CMRA	Simple clock	SMRA	SMRA-Clock-Hybrid	Clock+
Efficiency*	✗ Risk of missing bids and mistaken expectations	✓✓ Supports fluid switching between portfolios	✓✓ Supports fluid switching between portfolios	✓ Supports fluid switching between portfolios, but has risk of unsold lots unless accompanied by provisions for making exit bids	✓ Supports switching between portfolios but with some impediments	✓✓ Supports switching between portfolios minimising switching impediments	✓ Supports switching between portfolios but with some impediments that will manifest as incomplete switches
Realistic revenue**	✓ Limited incentives for strategic demand reduction	✓ No incentive for strategic demand reduction, but reliant on bidders being willing and able to express value for additional spectrum	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices	✓ Limited incentives for strategic demand reduction with appropriate reserve prices
Simplicity***	✗ Rules are simple, but decision making is strategically complex	✗ Complex rules and uncertainty over final liability (especially in the CCA) makes managing budget constraints difficult	✗ Complex rules, potentially complex bid decisions	✓✓ Simple rules and simple bid decisions	✓✓ Simple rules and simple bid decisions	✓✓ Simple rules and simple bid decisions	✗ Complex rules and uncertainty over what parts of a bid will be accepted
Transparency	✗ Substantial uncertainty over outcomes and no visibility of other bidders' decisions	✗ Uncertainty over outcomes especially in the CCA; calculation of discounts in ECCA not transparent	✓ No uncertainty over exposure and full control of own outcome, but process of establishing price increments may not be fully transparent	✓✓ No uncertainty over exposure and full control of own outcome	✓✓ No uncertainty over exposure and full control of own outcome	✓✓ No uncertainty over exposure and full control of own outcome	✓ No uncertainty over exposure, but potential difference between bids places and processed bid
Freedom of choice*	✓(✗) Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids without indication of winnable portfolios	✓ Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids	✓ Bidders can express demand for different portfolios, but potentially limited by constraints on number of bids	✓ Bidders can express demand for preferred portfolio at round prices	✓ Bidders can express demand for portfolios, potentially limited by standing high bids	✓ Bidders can express demand for portfolios, potentially limited by standing high bids	✓ Bidders can express demand for portfolios, but bids may not be accepted fully

* Assuming that complementarities between lots are limited and that there are no material aggregation risks.
 ** Assuming reserve prices are set close to prices expected in a competitive auction.
 *** Both mechanical and strategic.
 ✗ - does not achieve objective; ✓ - performs well in achieving objective; ✓✓ - performs very well in achieving objective.

Annex B Draft auction rules

B.1 Overview of award process

The award of spectrum in the 700, 1400 and 2100 MHz bands will consist of two stages.

- The first stage (the 'lot assignment stage') will determine the total bandwidth to be assigned to each bidder in each band. This stage will use a SMRA-Clock-Hybrid auction format.
- The second stage (the 'frequency assignment stage') will determine the specific frequencies to be assigned to each winner of lots. This stage will use a combinatorial second price auction.

The frequency assignment stage will only be held if there is more than one winner in the lot assignment stage for whom more than one assignment option exists and will only include those bands (or combination of bands) for which these multiple assignment options exist. If there is only one winner with assignment options in a frequency band, this winner will be able to request a specific frequency range that corresponds to the bandwidth it has been assigned in that band (or that combination of bands) and which lies either at the top or at the bottom of the band, leaving any unassigned spectrum as a contiguous block at one end of the band.

B.2 Available spectrum

The available frequencies are:

- 2x30 MHz in the 700 MHz band (703-733 MHz paired with 758-788 MHz), divided into six blocks of 2x5 MHz;
- 40 MHz in the 1400 MHz band (1452-1492 MHz, designated for supplementary downlink), divided into eight blocks of 5 MHz; and
- 2x60 MHz in the 2100 MHz band (1920-1980 MHz paired with 2110-2170 MHz), divided into twelve blocks of 2x5 MHz.

In the lot assignment stage, the available spectrum will be offered in the form of frequency-generic lots, with each band forming a separate lot category.

The table below shows the lot categories, the size of individual lots, the number of lots available and the reserve price per lot.

Lot Category	Size	Number of lots	Reserve price per lot
700 MHz FDD	2x5 MHz	6	[XXX]
1400 MHz SDL	5 MHz	8	[XXX]
2100 MHz FDD	2x5 MHz	12	[XXX]

B.3 Spectrum caps

Bidders are constrained in the amount of bandwidth they can win in the lot assignment stage by the applicable spectrum caps. These are specific for each bidder and take account of the bidder's current spectrum holdings.

Specifically, the number of lots on which a bidder can place bids in the lot assignment stage is limited by the constraint that, if the bidder were to win all the lots included in its bid, it must not hold:

- more than 40% of all spectrum designated for the provision of mobile services (which at present comprises 700 MHz, 800 MHz, 900 MHz, 1400 MHz, 1800 MHz, 2100 MHz, and 2600 MHz);
- more than 40 % of spectrum below 1 GHz designated for the provision of mobile services, rounded up to the nearest multiple of 10 MHz (2x5 MHz).

This constraint will apply going forward and include any further band designated for the provision of mobile services.

The auctioneer will determine the auction-specific spectrum cap for each qualified bidder based on the bidder's existing spectrum holdings.

B.4 The application process

[Detailed rules for the application and qualification process to be set by AT, including deposit requirements etc. This must include a requirement for applicants to specify their demand for spectrum in each lot category at reserve prices, which will establish the need for conducting the lot assignment stage.

Only qualified bidders will be permitted to proceed to the auction]

B.5 The lot assignment stage

B.5.1 Overview of the lot assignment stage

If it is possible to assign to all qualified bidders the lots they have specified in their application, then each bidder will receive the number of lots specified in the initial application and will be liable to pay the reserve price for each block, and the auction will proceed to the frequency assignment stage.

Otherwise, a bidding process will be used to determine the assignment of lots. The bidding process consists of one or more rounds in which bidders may submit bids for the lots available at prices announced by the auctioneer. At the end of each round, the auctioneer determines provisional winning bids in each lot category (the 'standing high bids'). The process ends after the first round in which no new bids or waivers are submitted. Standing high bids then become winning bids, and winners will then be required to pay for each of the lots they have won the price at which they placed their corresponding winning bid.

B.5.2 Bids

A bid is an offer to acquire a lot at the round price in the round in which the bid is placed.

In order to submit new bids, bidders need to indicate the number of lots they wish to acquire in the corresponding lot category at the prevailing round price. This establishes a commitment to acquire up to the number of lots in the specified lot category at the round price.

For the avoidance of doubt, a bidder may become standing high bidder, and win, a subset of the bids it has made and/or maintained in a given round.

A bid is only valid if it is submitted during a round in accordance with the auction rules set out below.

B.5.3 The bidding process

The bidding process involved one or more rounds, where each round is a fixed time window during which bidders may submit bids at the round prices announced by the auctioneer.

When scheduling a round, the auctioneer will announce, for each lot category, the price per lot that will prevail in the round (the 'round prices');

While the round is in progress, bidders may specify the number of lots in each lot category for which they wish to submit a bid at given round prices.

Bidders may not bid at a price that differs from round price.

Scheduling of rounds

Rounds will be scheduled at the auctioneer's discretion.

When a round is scheduled, the following information will be made available to each bidder:

- the start and end time of the round;
- the round price for each lot category;
- its own eligibility level (explained below);
- the number of waivers it has left (explained below); and
- the standing high bids it holds at the start of the round;

Bid submission during a round

In each round, bidders can make a single submission of bids using the method prescribed by the auctioneer.

To make a submission, a bidder will need to specify, using the bid form provided, the number of lots in each category for which it wishes to submit a bid at the round prices (subject to the constraints on valid bids set out below).

In the first round, bidders are required to submit at least one bid, and are not allowed to use waivers. Any bidder who fail to make a submission in the first round will be lose its eligibility to continue to bid in the auction.

Waivers will be used as the default submission for bidders who have waivers left and fail to make a submission in a round (after the first round) in which their activity from standing high bids is below their eligibility. Bidders can avoid using a default waiver in these cases by making a submission (including the possibility to submit an empty bid form if the bidder does not wish to submit any bids in the round).

Details of the bid submission process will be set out in the appropriate auction manuals, which will be provided to qualified bidders. The submission process is only completed when the bidder has received confirmation that its bid has been received.

Activity rules

The activity of a bidder in a round cannot exceed the bidder's eligibility for that round.

The activity of a bidder in a round is calculated as:

- the number of lots for which the bidder holds a standing high bid in lot categories where the bidder does not submit any bids; plus
- the number of lots for which the bidder submits bids in the round.

The eligibility of a bidder in the first round is equal to the initial eligibility determined by the bidder's application.

In each subsequent round, the eligibility of a bidder will be equal to:

- if the bidder has used a waiver in the preceding round, its eligibility in the preceding round;
- otherwise, its activity in the preceding round.

Each bidder may use up to **[three]** waivers during the auction (though the auctioneer can, at its discretion, give additional waivers to bidders).

The effect of the waiver will be to preserve the eligibility of the bidder for the following round; thus, eligibility reductions are only made in relation to the bidder's activity in a round in which it does not submit a waiver.

Bidders cannot use a waiver in the first round.

Valid bid combinations

A bidder may only submit a combination of bids such that, if the bidder were to win all the bids submitted along with any standing high bids it may hold, the bidder would not breach any applicable spectrum caps.

Bidding for lots when then bidder holds standing high bids

After the first round, a bidder that holds standing high bids in a lot category may submit bids in that lot category provided that the number of lots specified in the bid is:

- at least as large as the number of lots on which the bidder holds standing high bids if the current round price is above the round price at which the standing high bids have been placed; and
- strictly larger than the number of lots on which the bidder holds standing high bids if the current round price is unchanged from the round price at which the standing high bids have been placed.

Therefore, bidders are only allowed either to maintain their existing standing high bids or to increase their demand relative to their standing high bids.

For example, a bidder who holds standing high bids for x lots in a category at price p may only submit bids in that category for:

- x or more lots if the round price is greater than p ; or
- more than x lots if the round price remains at p .

New bids will replace the bidder's earlier bids in the respective category and will compete on equal terms with the bids from other bidders for becoming standing high bids.

B.5.4 Round prices

In each round, the auctioneer will specify the round price per lot for each lot category. Round prices will be specified in whole thousands of Euros.

In the first round, the round price for each lot category will be the reserve price per lot in that category.

In subsequent rounds, the round price will increase for lot categories in which all lots have a standing high bid at a price equal to the round price in the most recent round. Otherwise, the round price will remain unchanged.

Therefore, round prices may not decrease over the course of the auction.

The increase in round prices, when applicable, will be determined at the auctioneer's discretion and may vary across lot categories and across rounds. However, it is not expected that round prices will be increased by more than **[15%]** or by less than **[2%]**.

B.5.5 Determination of standing high bids

At the end of each round, the auctioneer will determine the standing high bids for each lot category. Standing high bids are determined for each lot category independently.

The process for determining standing high bids for a lot category is based on establishing a queue of the bids received for each lot category and designating a number of bids no greater than the number of lots available in that category as standing high bids, selecting bids in order from the queue. The queue is formed, separately for each lot category, as follows:

- bids submitted in the current round are placed first in the queue, ordered by bidder with the order of bidders determined at random;
- next, if there are any standing high bids for lots in that lot category from the preceding round which have been maintained in the current round (i.e. from bidders who have not made new bids in the round), then these bids will be

added to the back of the queue in the same order as in the preceding round.

For each lot category the auctioneer will then select the bids in the order as they have been queued as the standing high bids for the corresponding lot category, selecting a number of bids that is equal to the smallest of (i) the supply of lots in that lot category; and (ii) the number of bids in the queue.

As a consequence of these rules, at most one standing high bidder in each lot category may hold standing high bids on fewer lots than it bid for in that category in the round in which it submitted these bids, if the number of bids in the queue for that lot category exceeds the number of lots available in that category.

The following example illustrates the process.

Suppose that we receive bids from three bidders (A, B and C) for blocks in the 700 MHz band where six lots are available. Suppose that A bids for four lots, B for three lots and C for two lots.

We first rank bidders A, B and C at random. Suppose we obtain the ranking B, A, C. This produces the following queue:

B	B	B	A	A	A	A	C	C
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Supply

As the auctioneer can accept at most six bids, the bids highlighted in bold are identified as standing high bids.

B.5.6 End of the lot assignment stage bidding process

The bidding process ends after the first round in which no new bids, or waivers are submitted.

B.5.7 Determination of winning bids

At the end of the lot assignment stage bidding process, standing high bids will become winning bids.

The base price for each winning bid will be equal to the round price at which the bid was submitted.

B.5.8 Information released at the end of each round of the lot assignment stage bidding phase

At the end of each round, bidders will be informed about whether a further round is needed or whether the lot assignment stage has ended.

If a further round is needed, the bidder will be informed about:

- the bids submitted and the standing high bids held by the bidder in each lot category;
- the bidder's eligibility for the next round;
- the number of waivers remaining for the bidder;
- for each lot category, aggregate demand (expressed in terms of the total number of lots included in newly placed and maintained standing high bids in that category); and
- the new round prices for each lot category.

If no further round is needed, the bidder will be informed about the number of lots it has won in each lot category, and the total price payable.

B.6 The frequency assignment stage

B.6.1 Overview of the frequency assignment stage

Only bidders who have won frequency lots will be able to participate in the frequency assignment stage.

The frequency assignment stage consists of a single round in which bidders will be able to place bids on the assignment options that are available to them in each of the bands included in this stage.

Bids will be placed simultaneously for all of the bands/band combinations included in this stage but evaluated separately for each of these.

B.6.2 Assignment options

Assignment options will be determined by the auctioneer for each winner of lots. Assignment options will be determined jointly for the 700 and 1400 MHz bands, and separately for the 2100 MHz band and will comprise all possible assignments of specific frequencies to the bidder in the band(s) that meets the following conditions:

- The frequencies that would be assigned in the option are contiguous within the band.

- The number of frequency blocks assigned to each bidder is equal to the number of lots in that band which it has been assigned in the lot assignment stage.
- The assignment does not preclude the assignment of contiguous frequencies to other bidders or the retention of unsold spectrum as a contiguous block at the upper or the lower end of the band.

Each bidder is guaranteed to receive one of its assignment options in each band in which it has won lots, regardless of whether it places any assignment bids. Assignment bids only determine which bidder receives which assignment.

B.6.3 Bids

A bid is an offer to pay a price potentially up to the bid amount (in whole Euro) for being assigned the specific frequencies that correspond to an assignment option for that bidder. Each bid should indicate the maximum amount that the bidder would be willing to pay for receiving the frequency assignment associated with that option rather than another option.

Bid amounts may be zero but must not be negative.

B.6.4 The bidding process

Bidders will be able to submit bids for each of the assignment options available to them in the 700/1400 MHz bands and the 2100 MHz band through the means and within the time period specified by the auctioneer.

If a bidder does not place an assignment bid for an option, it will be deemed to have submitted an assignment bid for that option of zero.

B.6.5 Determination of winning bids and prices

Bids will be evaluated for the 700/1400 MHz bands and the 2100 MHz band separately.

In each case, the winning combination of bids is determined as the combination of bids with the highest value that can be satisfied, subject to the condition that the frequency assignments in these bids yield feasible band plans. A feasible band plan is defined by the requirement that assignments to different bidders must not be overlapping and that any unsold frequencies must be retained as a contiguous block at the upper or lower end of the band.

Should there be multiple combinations of bids with the same highest value that meet this condition, one combination will be chosen at random.

Each bidder will be required to pay a price for being assigned its winning options, which is calculated separately for the 700/1400 MHz band and the 2100 MHz band.

In each case, the prices that bidders will be required to pay for the frequency options they are assigned are calculated jointly for all bidders, using a second-pricing approach as follows:

We calculate the 'opportunity cost' for a subset of bidders in the band as the difference between:

- the greatest sum of bids from other bidders that could be achieved in any feasible band plan for that band; and
- the sum of winning bids for that band from other bidders.

The prices in the band are calculated jointly by applying the following conditions:

- the sum of individual prices⁴⁷ for each proper subset of bidders⁴⁸ cannot exceed the sum of their winning bids;
- the sum of individual prices for each proper subset of bidders⁴⁹ must be at least the opportunity cost for the subset;
- the sum of individual prices must be the smallest possible subject to prices satisfying the conditions above; and
- the sum of the squared differences between each bidder's individual price and its opportunity cost⁵⁰ must be the smallest possible across all prices that satisfy the conditions above.

These conditions yield a unique solution for the prices in each band.

⁴⁷ By 'prices' (in each band) we refer to a vector of prices with one price for each of the bidders, and by 'individual prices' we refer to the elements of this vector.

⁴⁸ Including all possible sets containing only some of the bidders and the sets containing each single bidder.

⁴⁹ As above.

⁵⁰ I.e. the assignment opportunity cost for the subset including only this bidder.